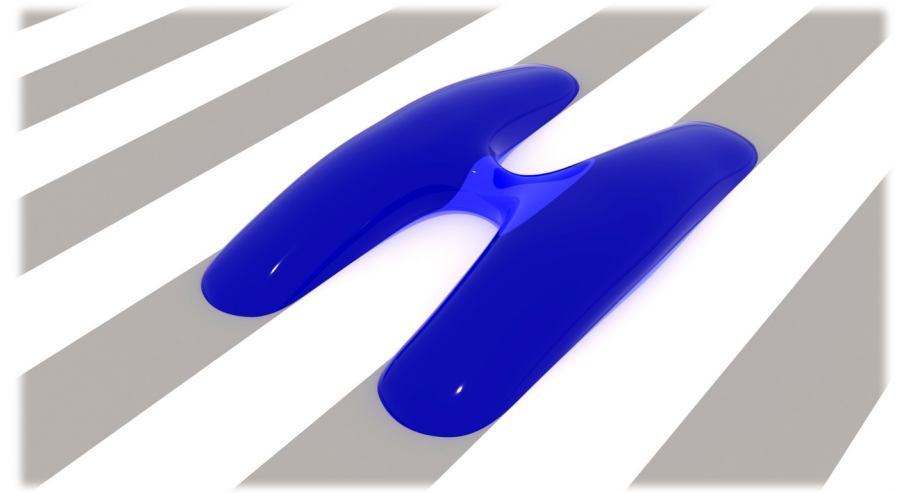


Wetting of Structured Surfaces: Modeling and Simulation

Mathis Fricke, Department of Mathematics,
Technical University of Darmstadt, Germany

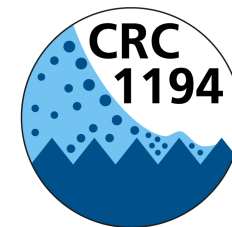
Conference on Functional Materials Engineering,
Hasselt, October **2024**



TECHNISCHE
UNIVERSITÄT
DARMSTADT



**Mathematical
Modeling and Analysis**



- Please feel free to **take pictures** of the slides!
- Publications with more details are referenced with **QR codes**.
- The slides are available on my webpage: www.mathis-fricke.de/presentations



Thanks to my collaborators ..



D. Bothe



T. Maric



J. De Coninck



S. Raju



Z. Tukovic



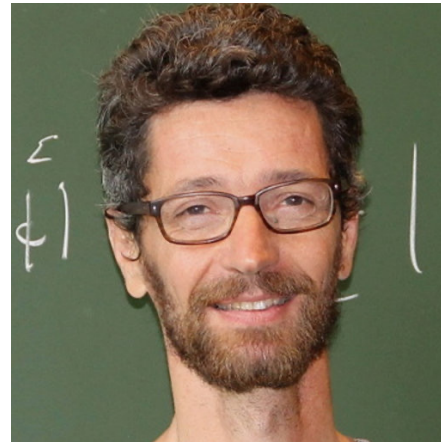
T. Fullana



Y. Kulkarni



S. Afkhami

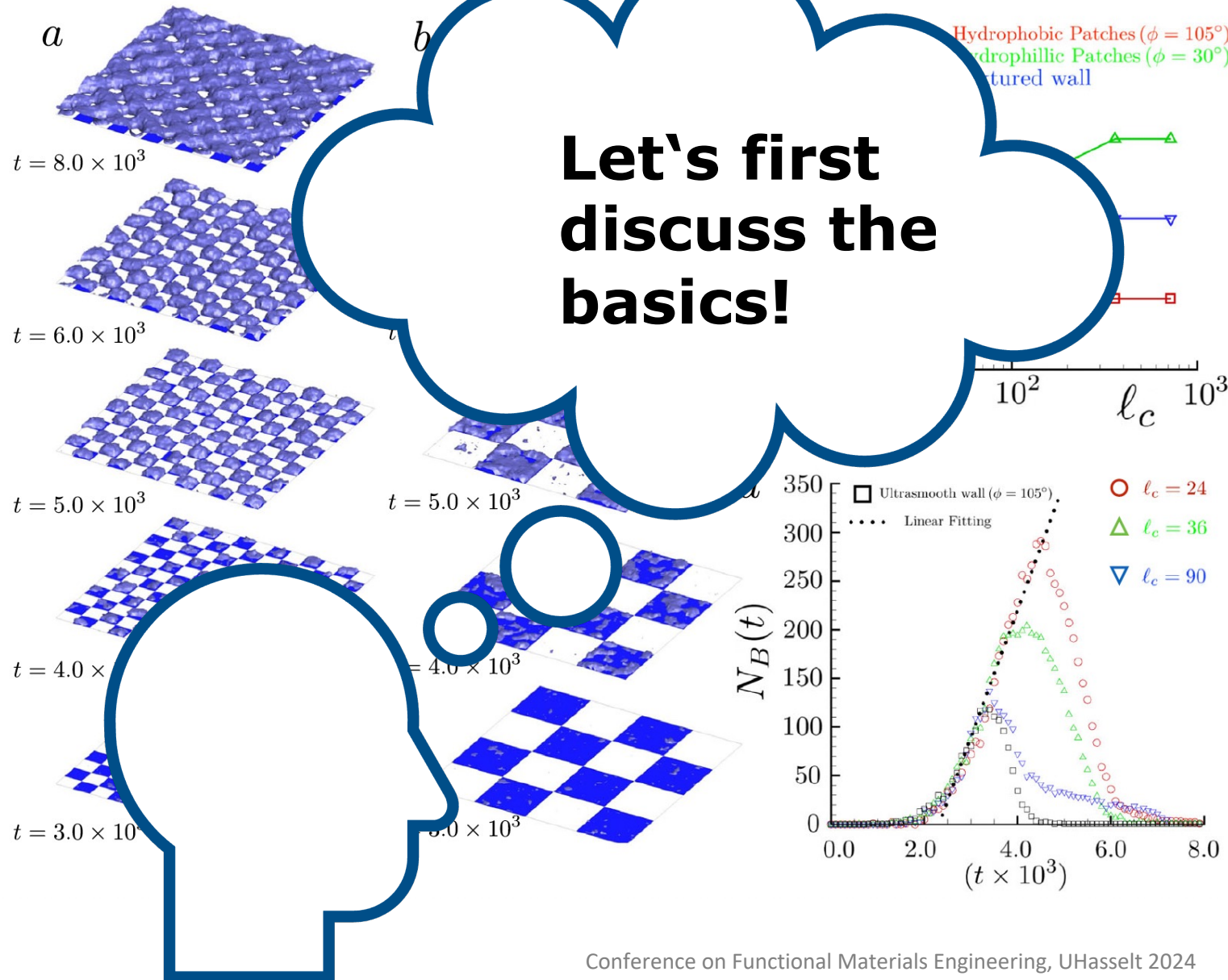


S. Popinet



S. Zaleski

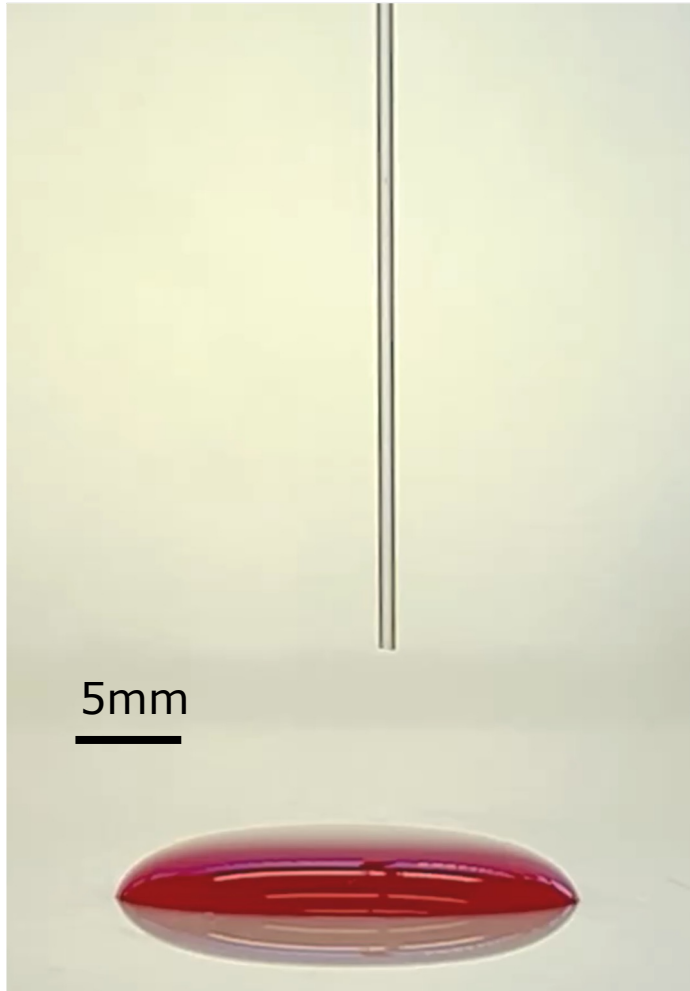
Recap: Surface Wettability and Structure can improve heat transfer



See yesterdays keynote by Joël De Coninck.

Gallo et al.,
 Nat. Commun. 14, **2023**

Capillary Rise: A prototypical dynamic wetting process



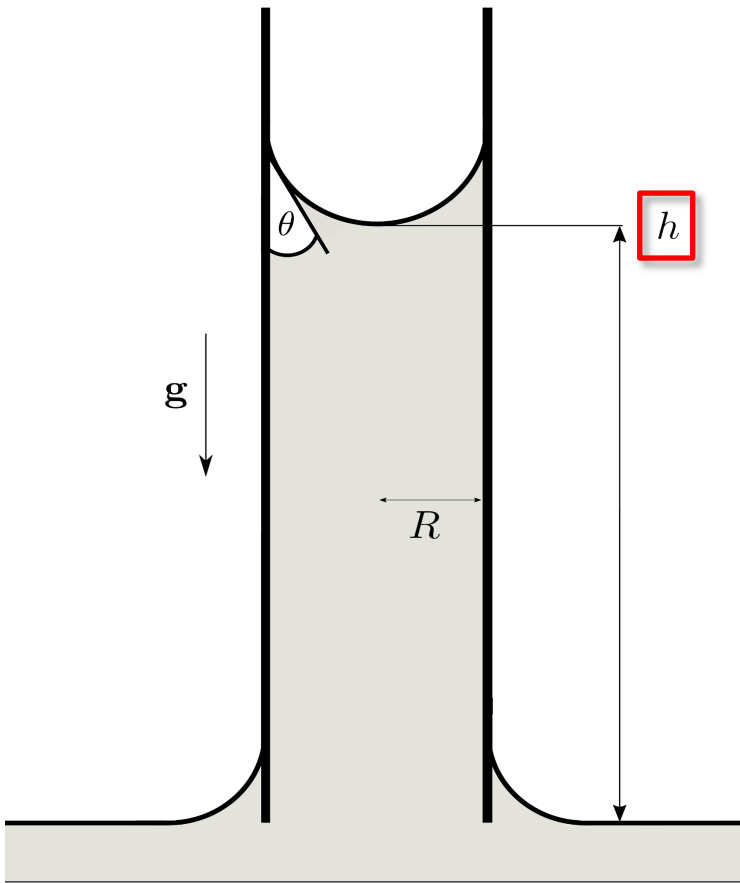
Why is this „simple“ process interesting?

- Numerous **applications** (e.g. porous media, microfluidics,...)
- **Prediction** of the dynamics is non-trivial!
- Axial symmetry → **Simplified models** available

Two major challenges

- **Predictive** models → only use well-defined material parameters
- **Multiscale** character → very high computational costs

Capillary Rise: A prototypical dynamic wetting process



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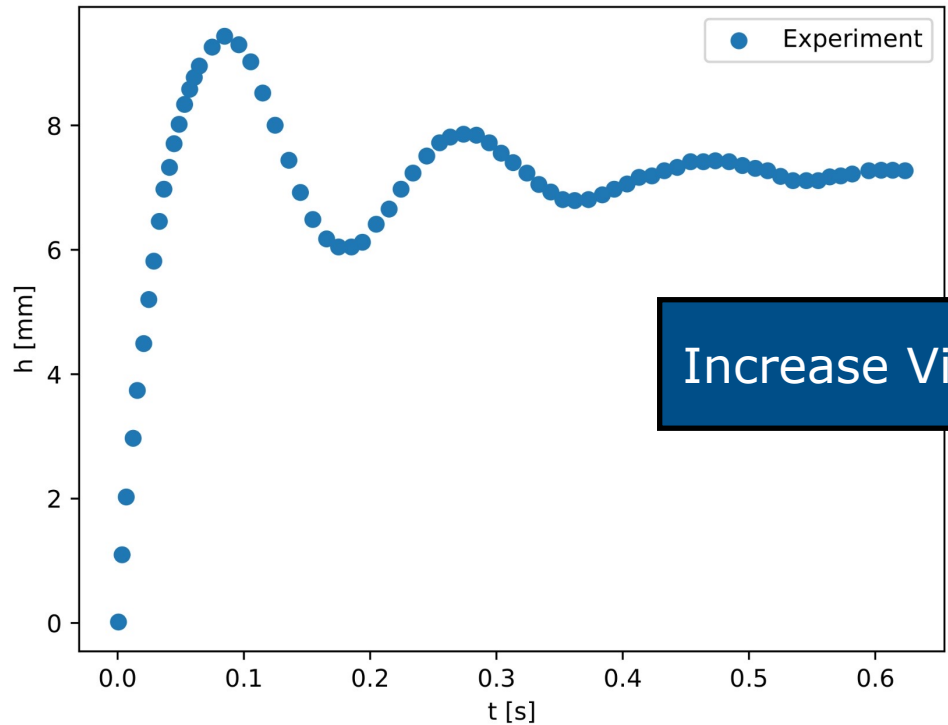
Two major challenges

- **Predictive** models → only use well-defined material parameters
- **Multiscale** character → very high computational costs

Goal: Predict rise height $h=h(t)$

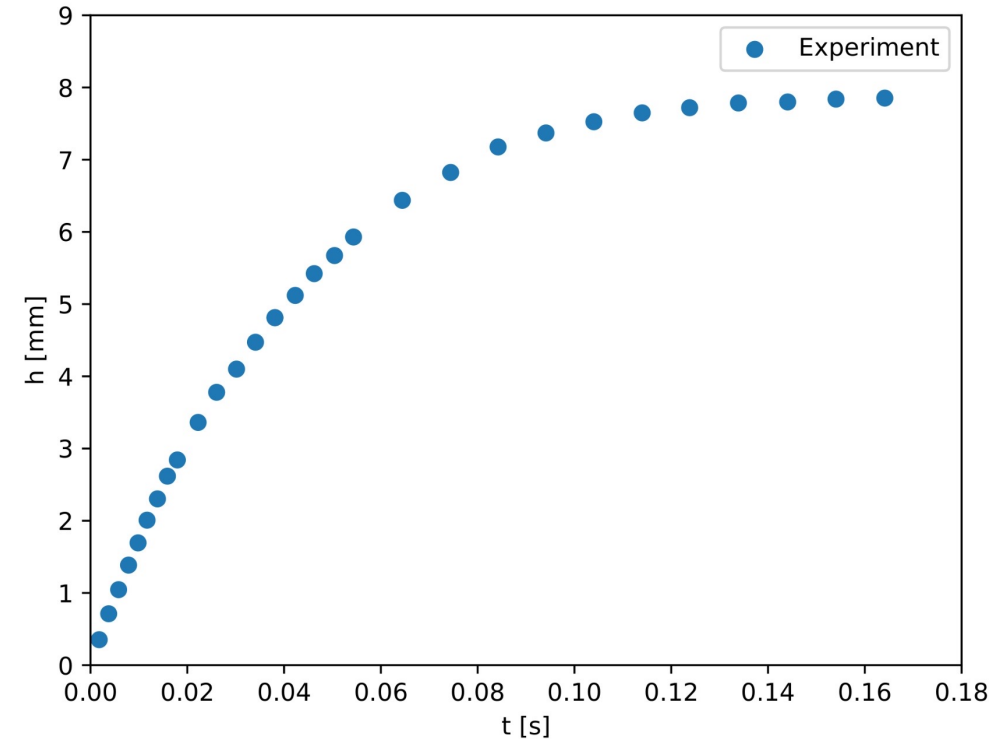
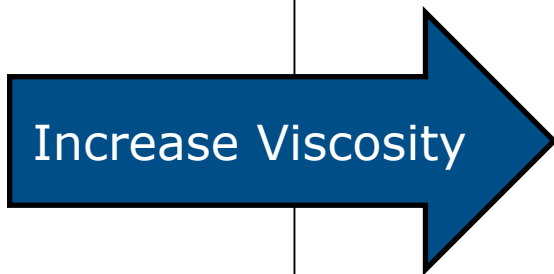
Experiments: Monotonic vs. Oscillatory Rise (Quéré, 1997)

- Experimental data for glass capillaries with radius $R = 689\mu\text{m}$.



Ether: $\eta = 0.3 \text{ mPa} \cdot \text{s}$

Low viscosity

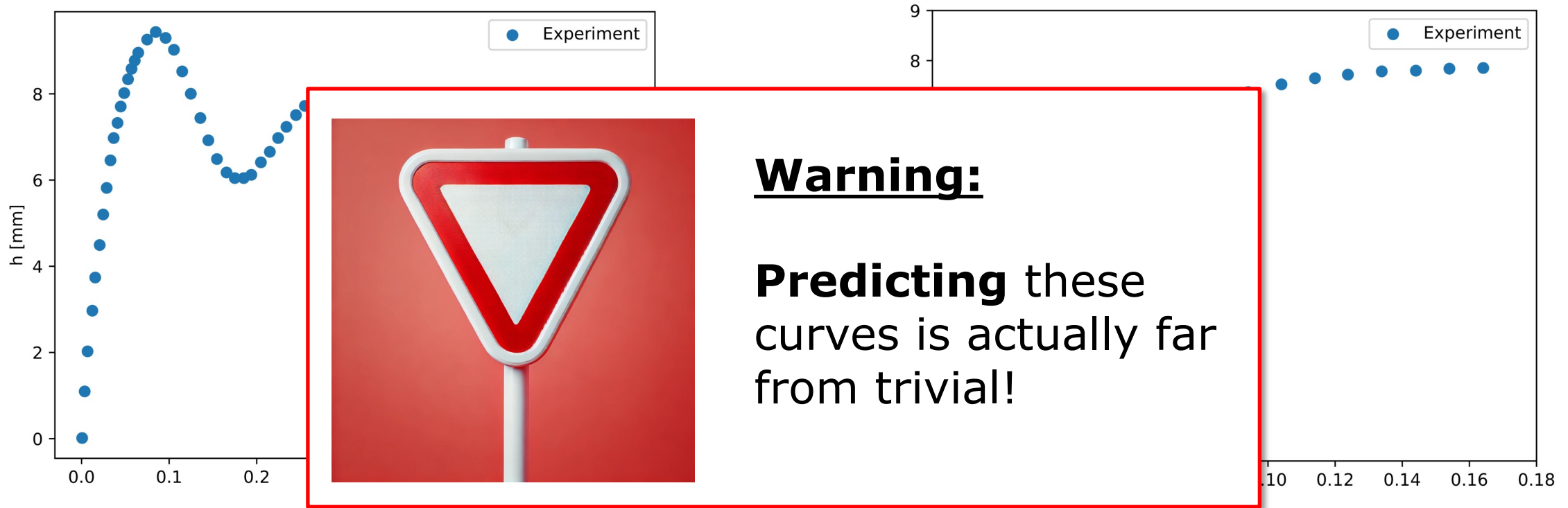


Ethanol: $\eta = 1.17 \text{ mPa} \cdot \text{s}$

High viscosity

Experiments: Monotonic vs. Oscillatory Rise (Quéré, 1997)

- Experimental data for glass capillaries with radius $R = 689\mu\text{m}$.



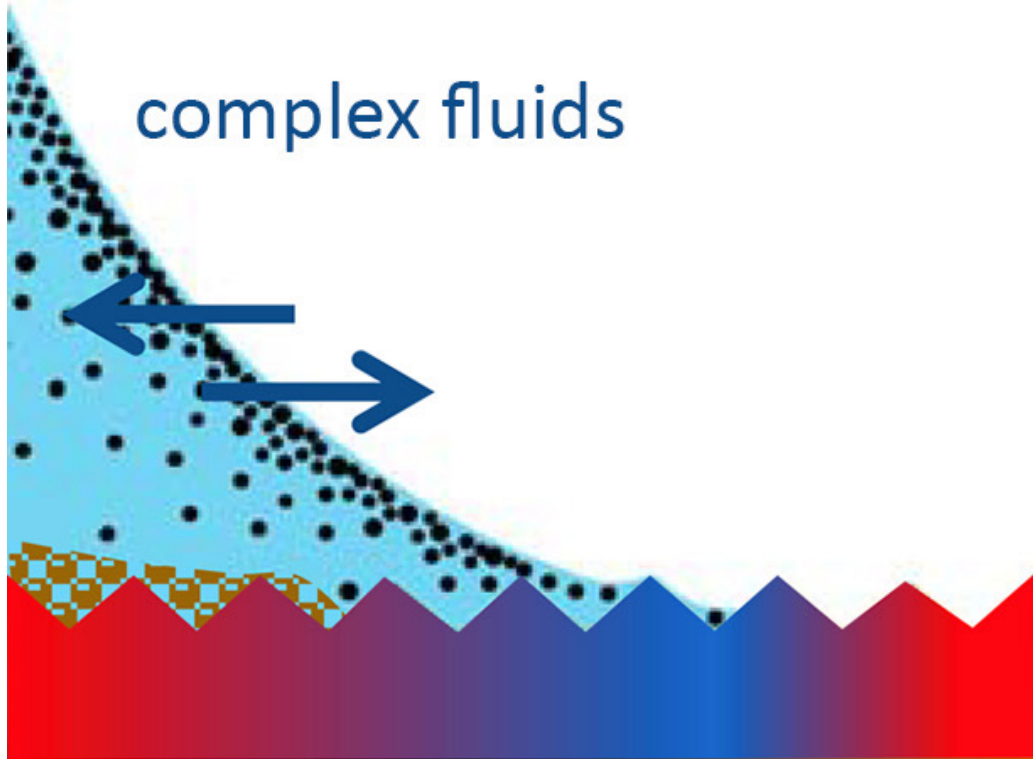
Ether: $\eta = 0.3 \text{ mPa} \cdot \text{s}$

Low viscosity

Ethanol: $\eta = 1.17 \text{ mPa} \cdot \text{s}$

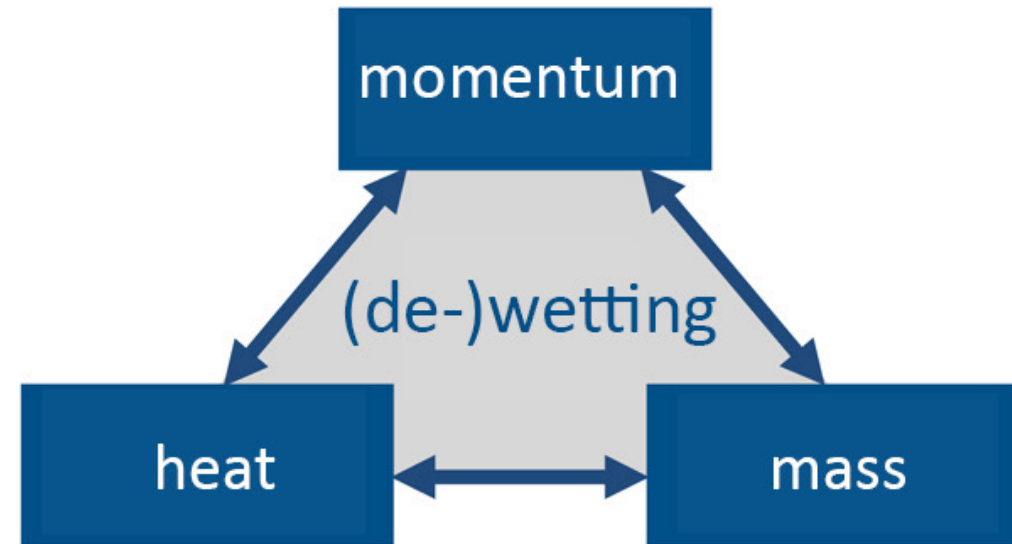
High viscosity

Collaborative Research Center on Wetting and Transport Phenomena

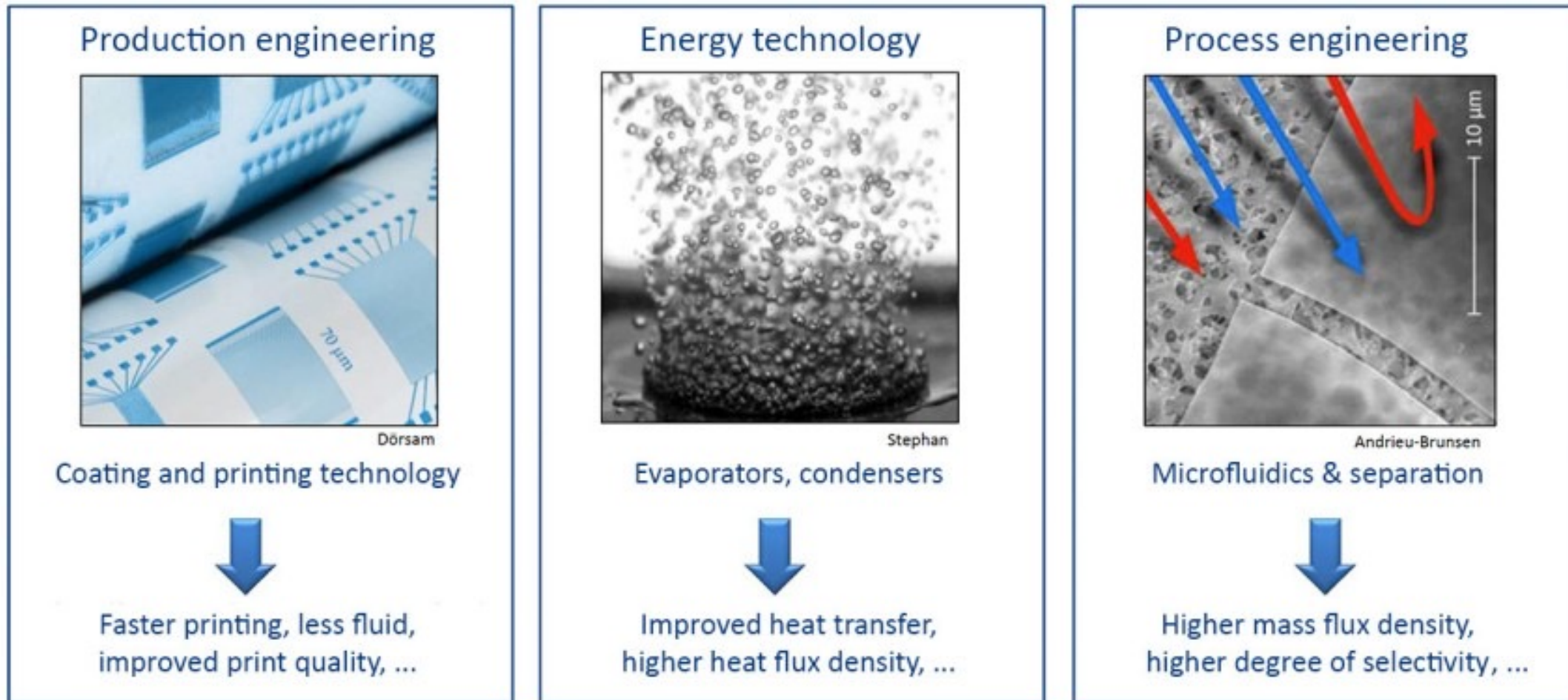


complex surfaces

transport/transfer

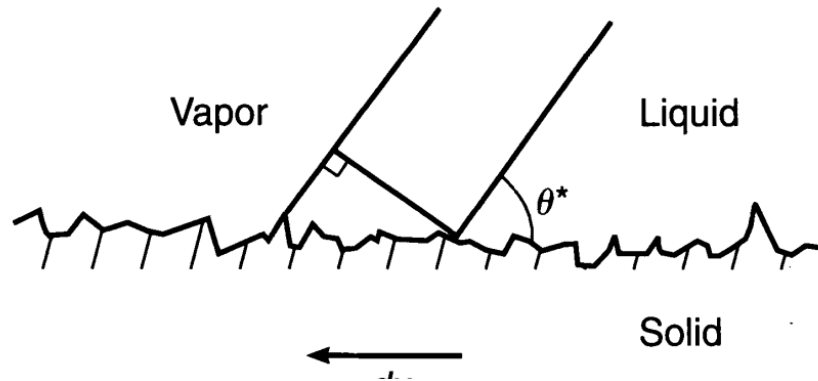


Collaborative Research Center on Wetting and Transport Phenomena



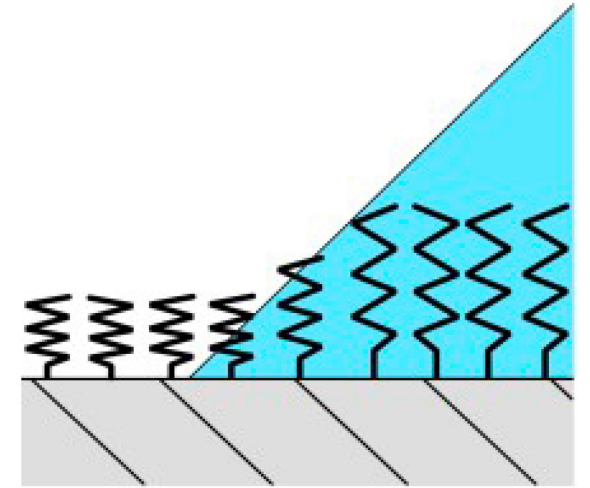
Examples: Wetting of complex surfaces

▪ Rough surfaces

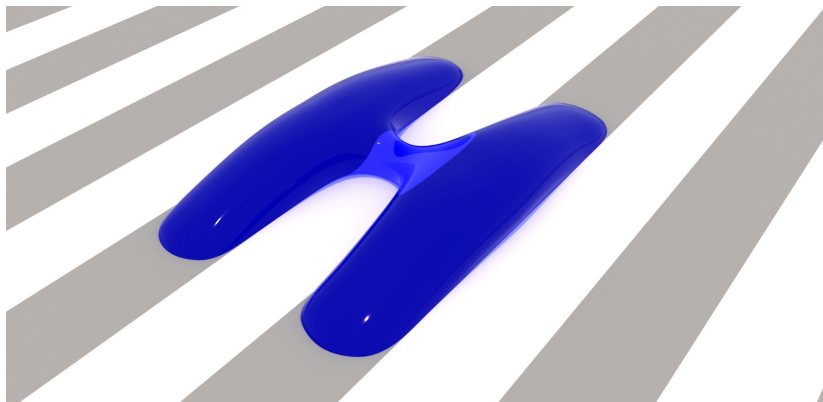


▪ Complex coatings (here Polymer Brushes)

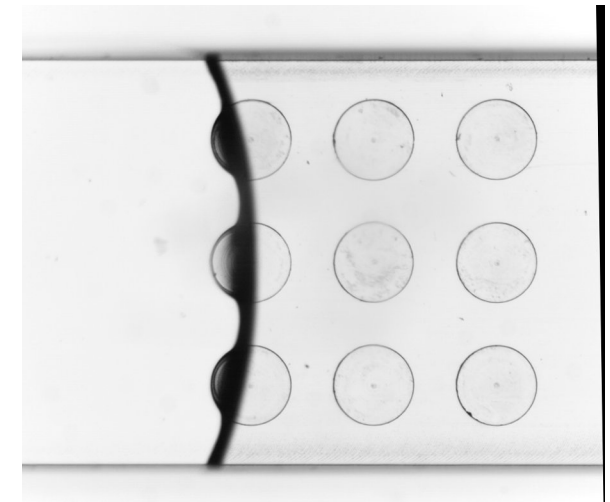
➔ **Adaptive wetting**



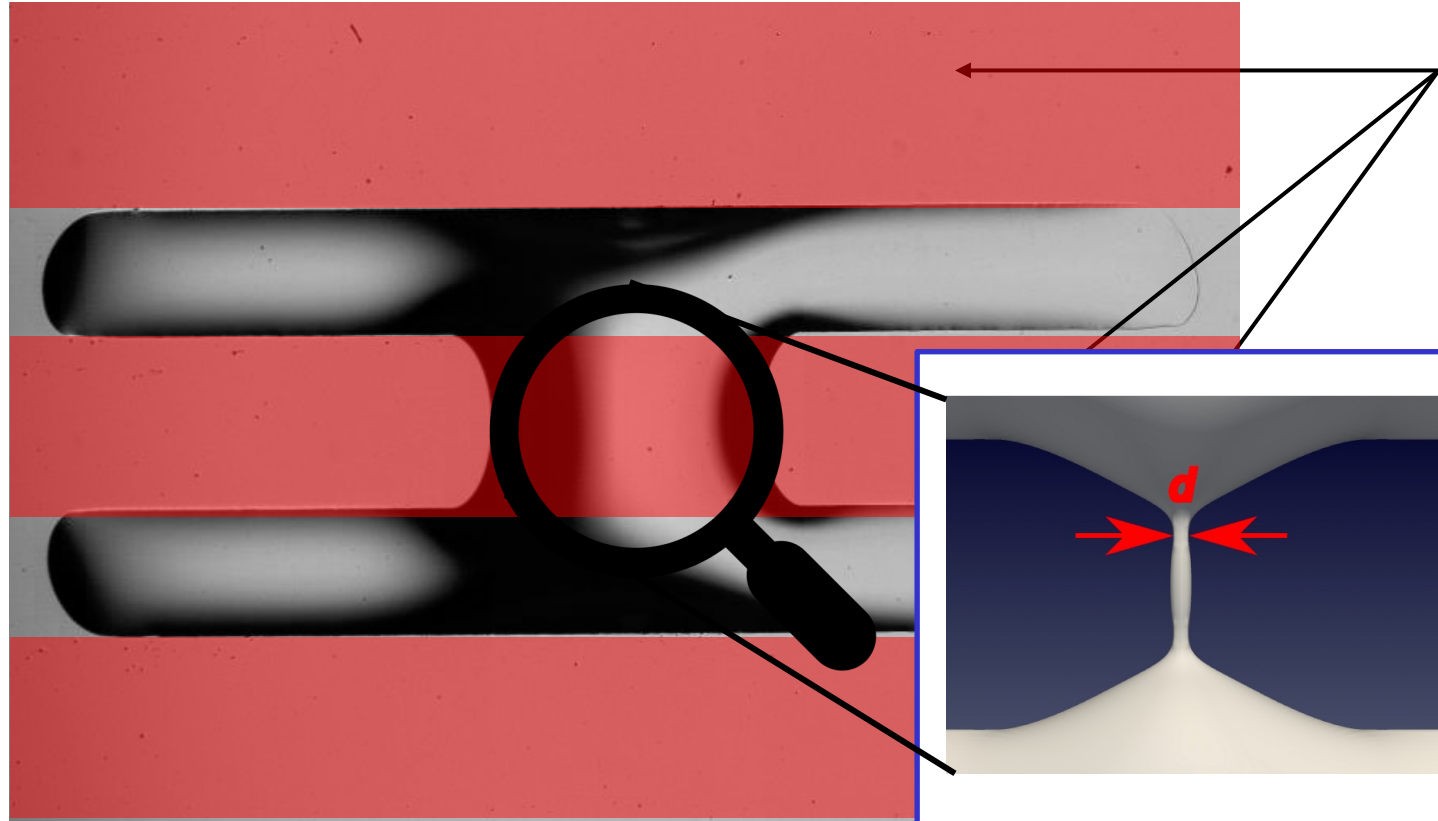
▪ Chemical heterogeneities



▪ Complex topography (here cavities)



Simulations give more insights – Breakup of a liquid bridge



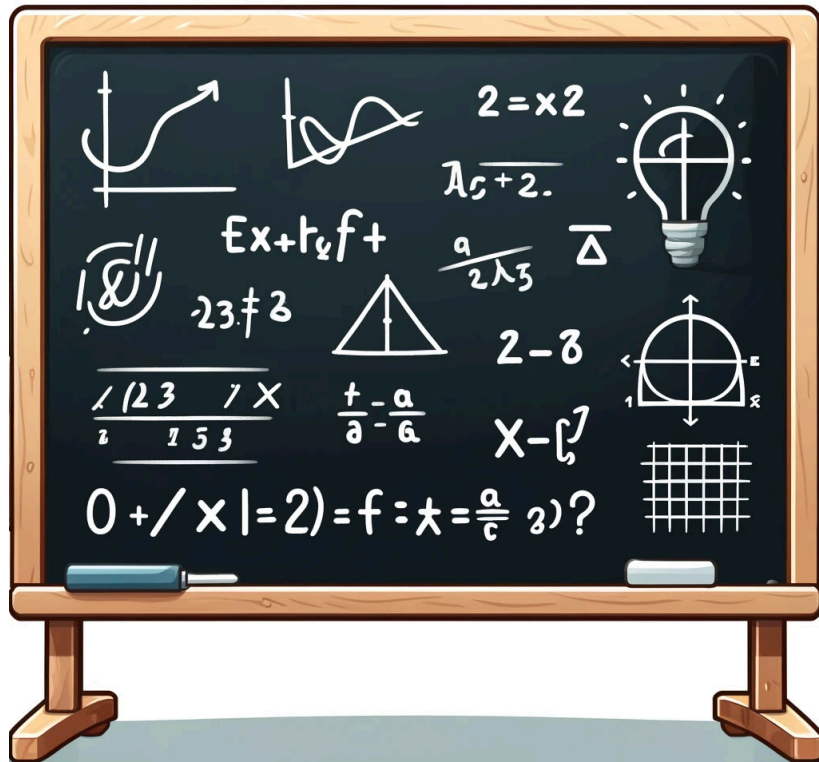
Structured surface:

Hydrophobic part of the substrate

Breakup is **too fast** for the high-speed camera!
Simulations help!

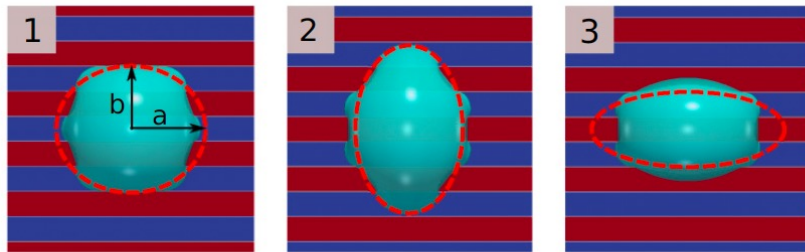
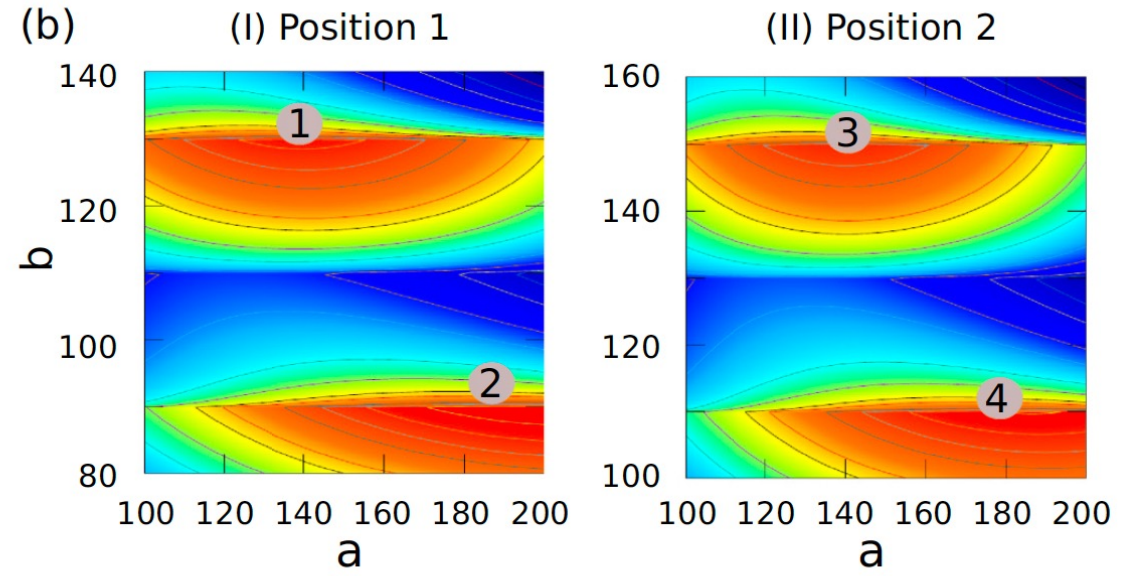
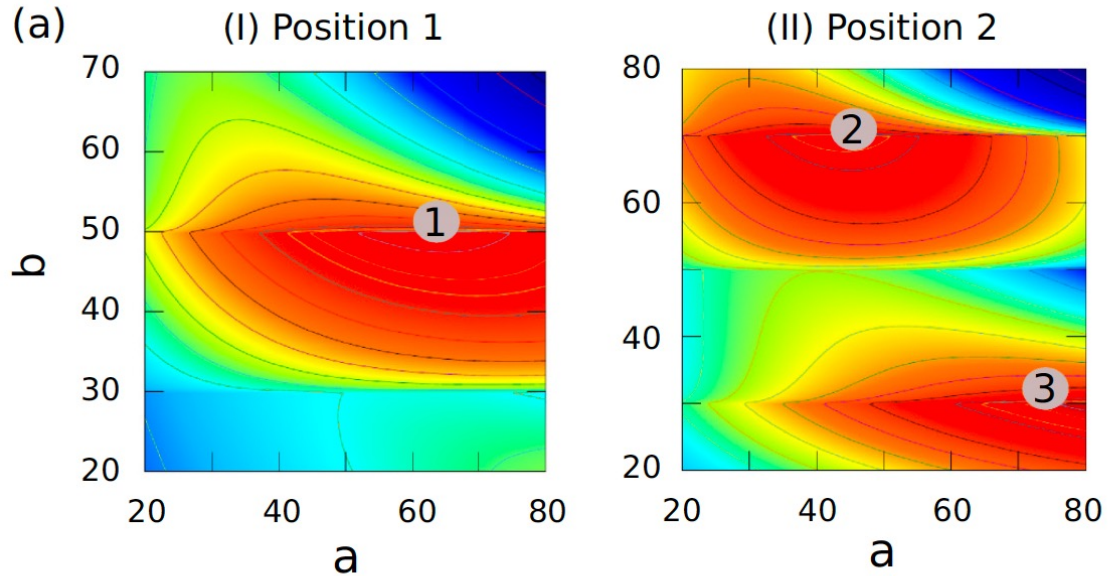


Hartmann, Fricke et al.: *Breakup dynamics of capillary bridges on hydrophobic stripes*, International Journal of Multiphase Flow 140, DOI: [10.1016/j.ijmultiphaseflow.2021.103582](https://doi.org/10.1016/j.ijmultiphaseflow.2021.103582), 2021

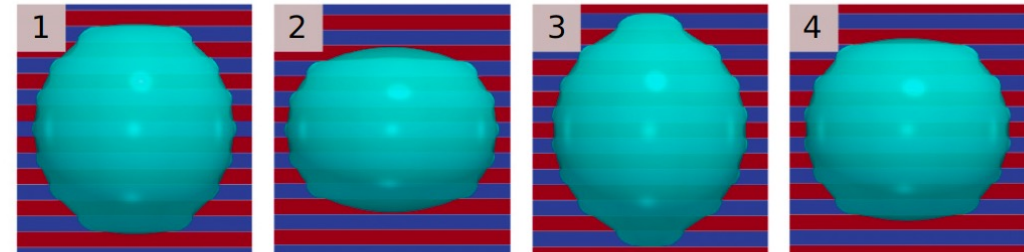


Modeling Fundamentals

Static Wetting \rightarrow Minimization of the free energy



(III) Snapshots



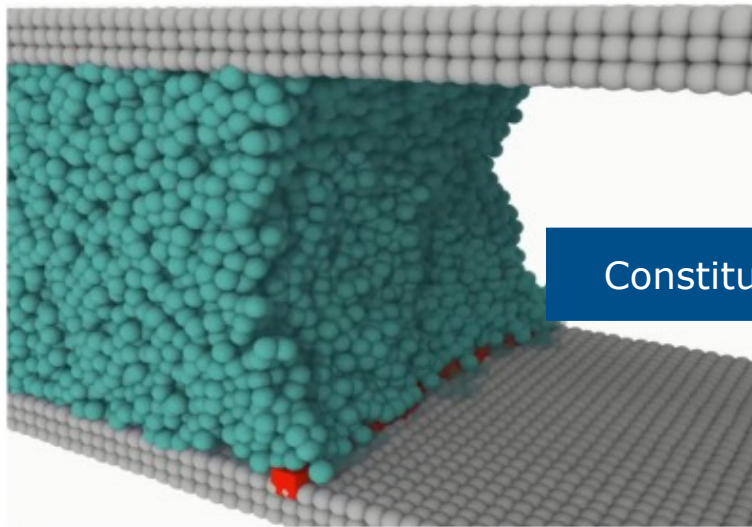
(III) Snapshots

Wu, Wang, Selzer and Nestler, Physical Review E 100, **2019**

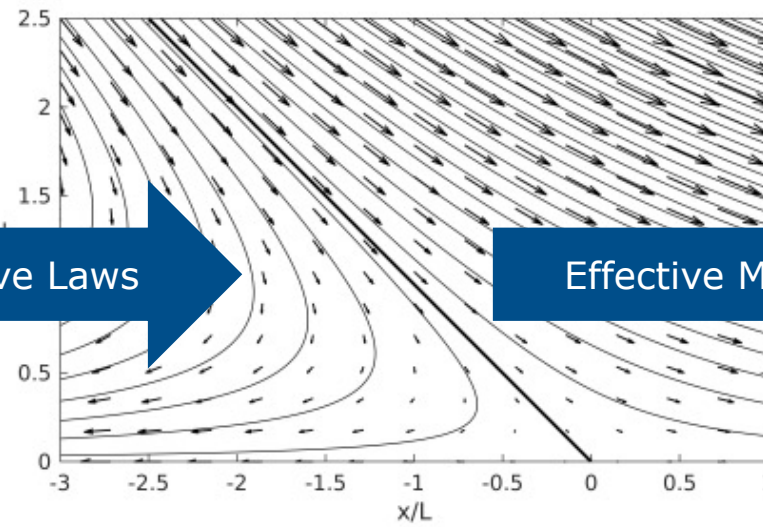


Modeling Hierarchy for Wetting Dynamics

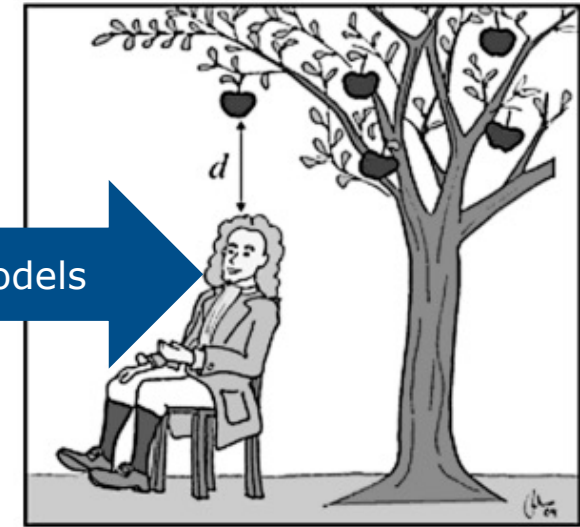
- **Molecular Dynamics (MD):** Modeling from first principles, highly accurate, limited in terms of time and length scales.
- **Continuum mechanics (Navier Stokes):** Inherits knowledge from MD in form of constitutive laws and boundary conditions, still costly.
- **Simplified models (typically ODEs):** Fast to solve, Taylor-made, basis for optimization!



(a) Molecular dynamics.



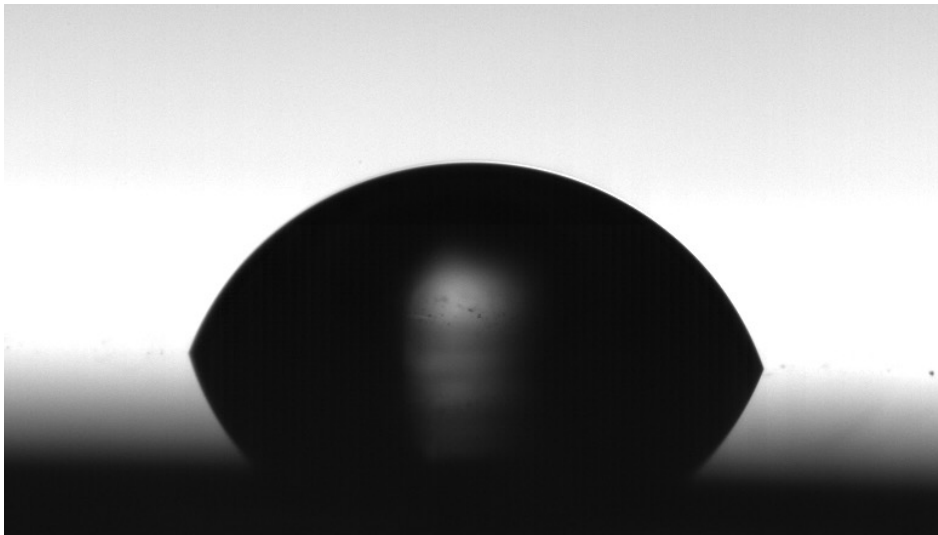
(b) Continuum mechanics.



(c) Simplified models.

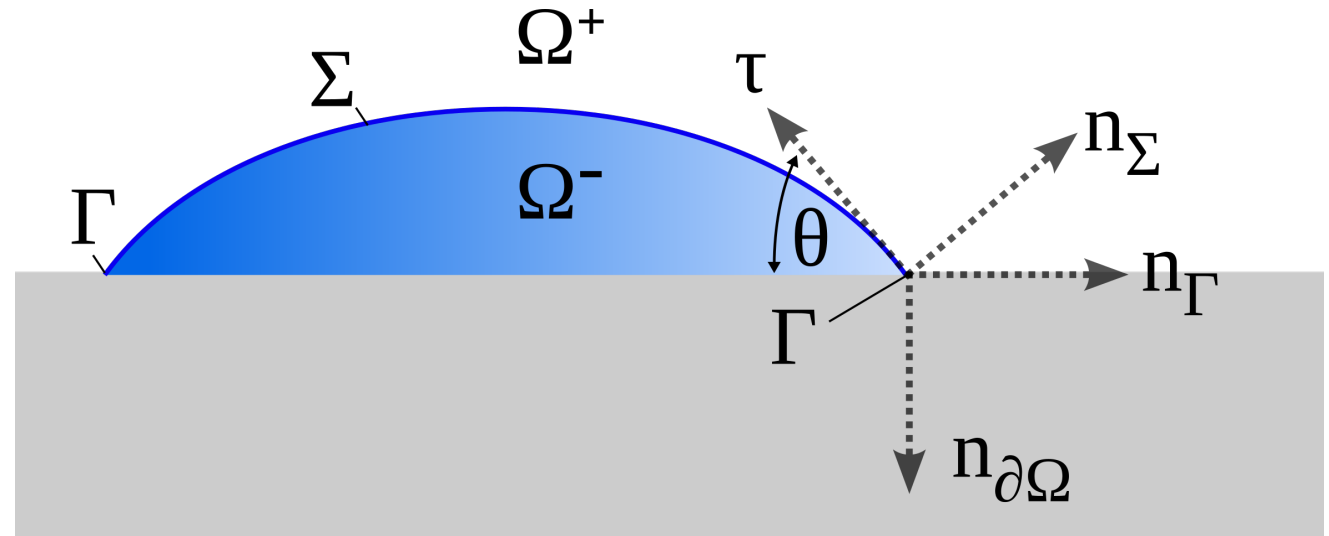
Complexity in Modeling & Simulation of Dynamic Wetting

- Real surfaces are **rough**,
- may have chemical or topographical **defects** or **structure**,
- Interfaces may be **contaminated** (e.g., surfactants).



Real droplet on a surface.

- **Simplify** the system as much as possible.
- Approximate interfaces as **sharp**.
- Treatment of surface defects with effective **boundary conditions?**
- **Initial conditions** sometimes unclear.



Mathematical model.

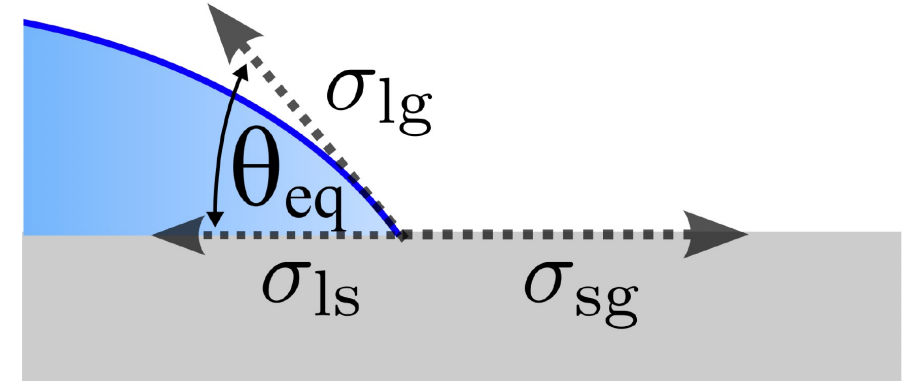
Static Wetting – The Young & Cassie-Baxter equation

- The contact angle on an **ideal** surface is governed by a balance of surface tension forces (Young equation):

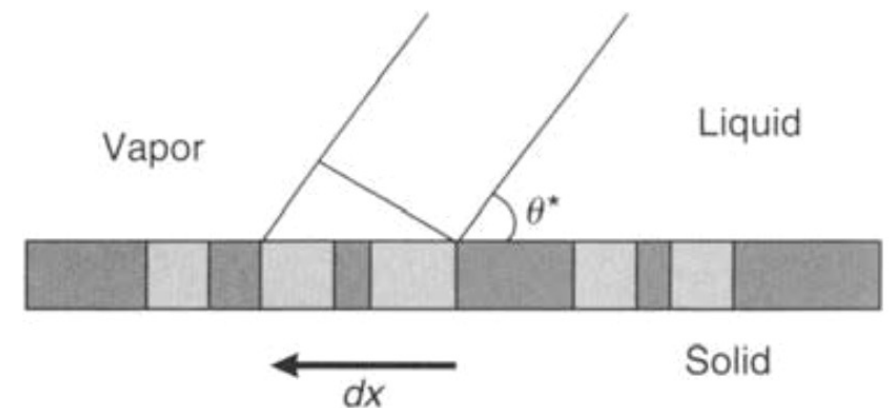
$$\sigma_{lg} \cos \theta_0 + \sigma_{sl} - \sigma_{sg} = 0$$

- Model for the **static** contact angle on **chemically textured** surface (Cassie-Baxter equation):

$$\cos \theta_{CB} = r_1 \cos \theta_1 + r_2 \cos \theta_2$$



Ideal surface

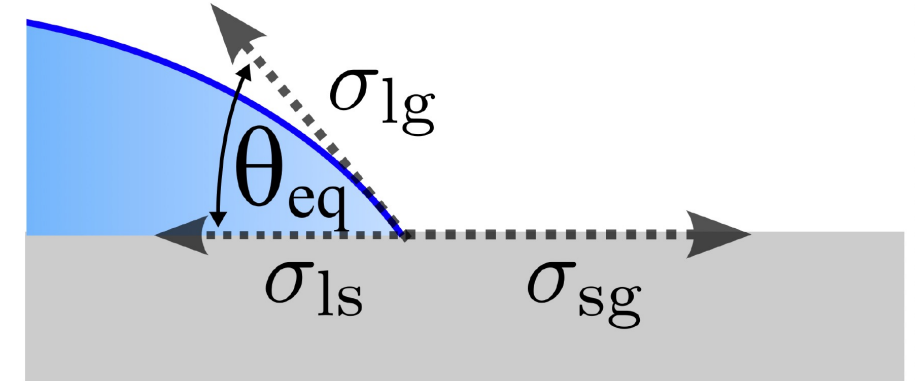


Chemically textured surface

Static Wetting – The Young & Cassie-Baxter equation



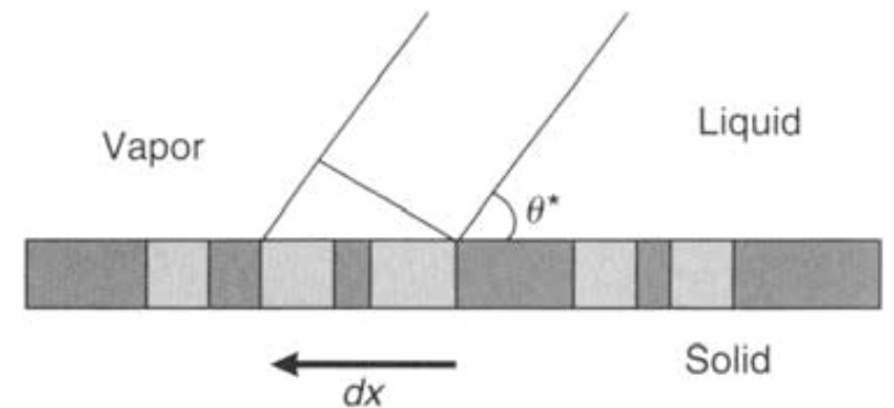
Can the Cassie-Baxter equation also be applied for **dynamic** cases?



Ideal surface

- Model for the **static** contact angle on **chemically textured** surface (Cassie-Baxter equation):

$$\cos \theta_{CB} = r_1 \cos \theta_1 + r_2 \cos \theta_2$$



Chemically textured surface

Two-phase Navier-Stokes Wetting „Standard Model“

(incompressible, Newtonian, isothermal, sharp-interface)

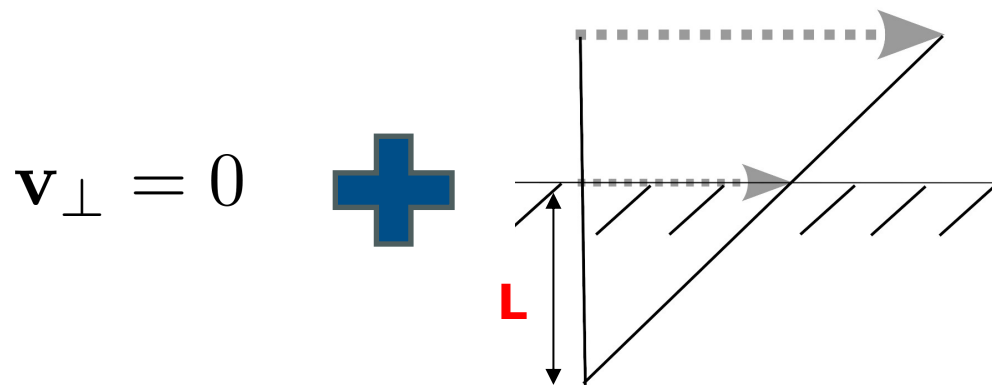
▪ Mass balance: $\nabla \cdot \mathbf{v} = 0$ ▪ Interface Kinematics: $V_{\Sigma} = \mathbf{v} \cdot \mathbf{n}_{\Sigma}$

▪ Momentum balance:

$$\partial_t(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla p = \eta \Delta \mathbf{v} + \rho \mathbf{g} + \mathbf{f}_{\Sigma}$$

▪ Surface tension force: $\mathbf{f}_{\Sigma} = \sigma \kappa_{\Sigma} \mathbf{n}_{\Sigma} \delta_{\Sigma}$

▪ Boundary conditions at the solid wall (slip!)



▪ Contact angle boundary condition:

$$-\zeta U_{cl} = \sigma (\cos \theta - \cos \theta_e)$$

Contact line friction: $\zeta \geq 0$

Contact line speed: U_{cl}

Two-phase Navier-Stokes Wetting „Standard Model“

(incompressible, Newtonian, isothermal, sharp-interface)

▪ Mass balance: $\nabla \cdot \mathbf{v} = 0$ ▪ Interface Kinematics: $V_\Gamma = \mathbf{v} \cdot \mathbf{n}_\Gamma$

▪ Mom

Recent developments in modeling:

Fullana, Kulkarni, Fricke et al.:

A consistent treatment of dynamic contact angles in the sharp-interface framework with the generalized Navier boundary condition, Preprint,

2024

DOI: 10.5281/zenodo.10142047

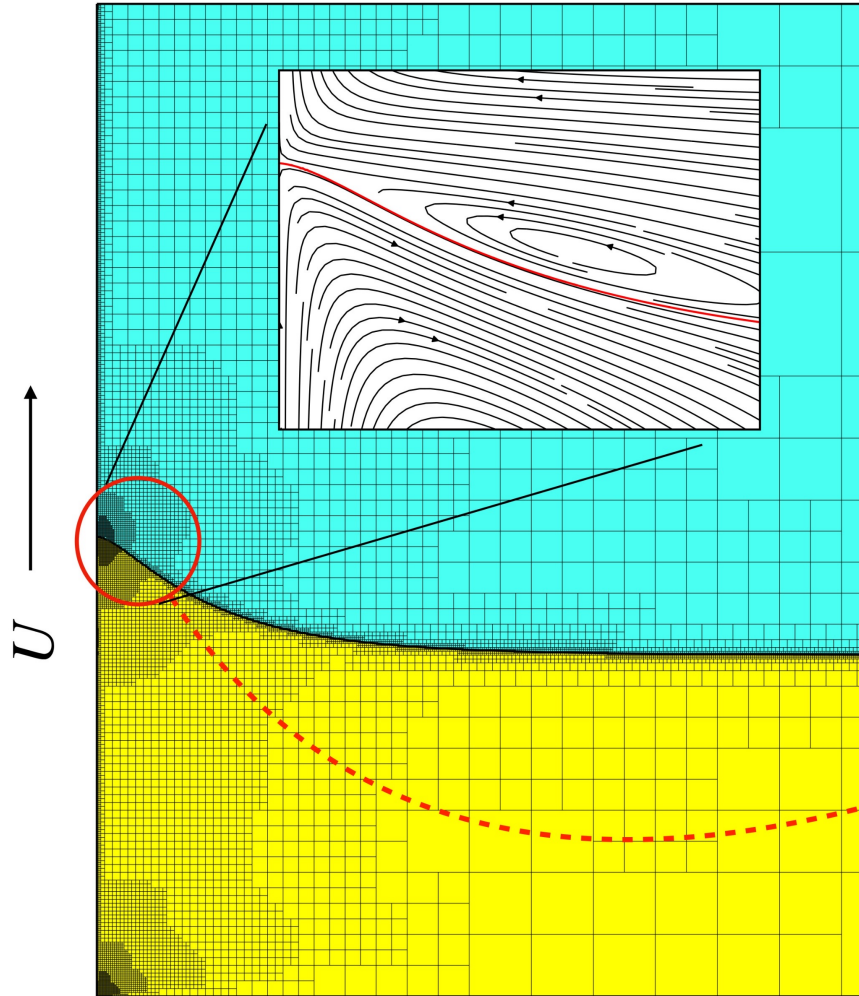


▪ Surface

▪ Bound

$\mathbf{v}_\perp =$

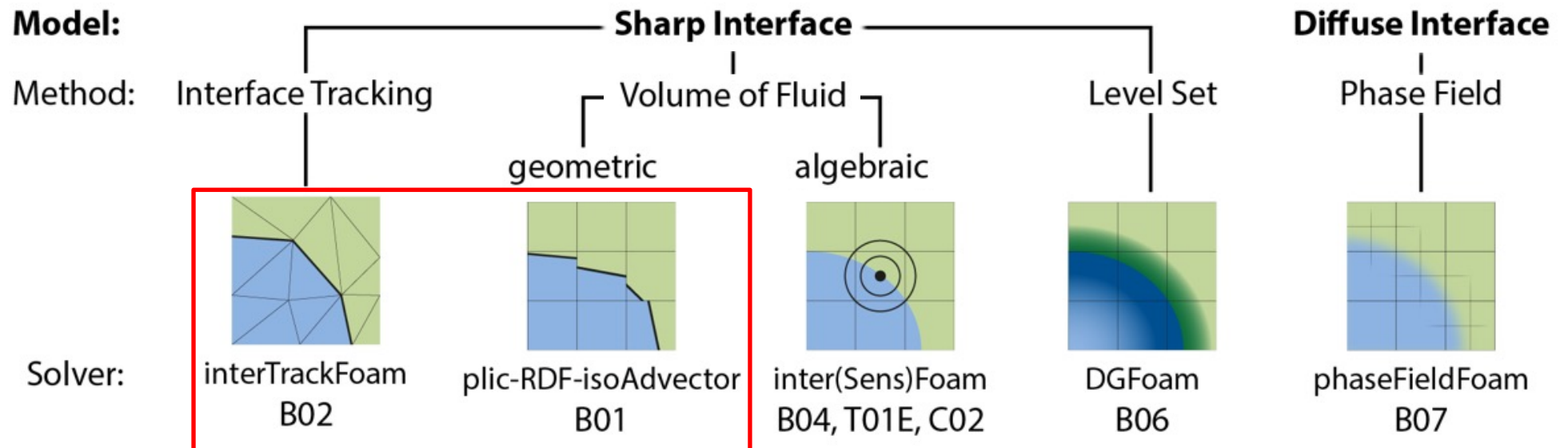
Contact line speed: U_{cl}



Numerical Simulation Fundamentals

Our approach to CFD simulation

- There is no single method suitable for every problem!
- We develop diverse numerical methods for two-phase flows in the **OpenFOAM** library.
- Specific methods to tackle, e.g., topological changes, surfactant transport, evaporation, ...

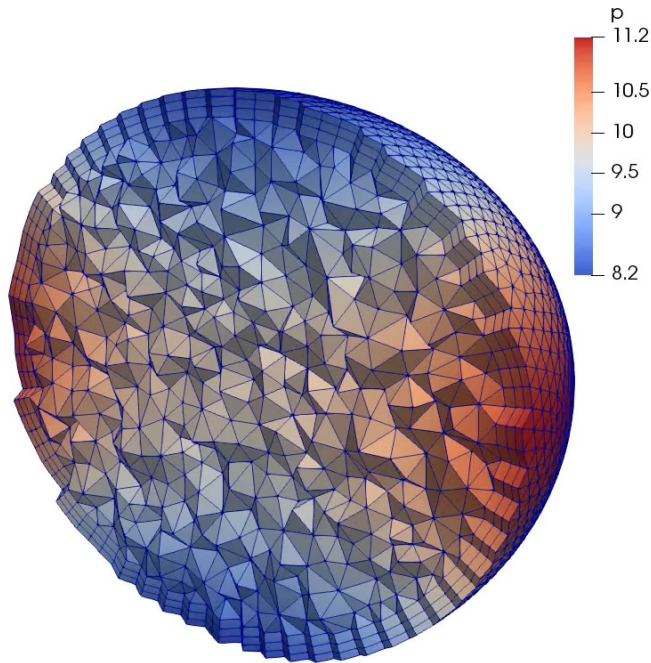


Tracking the fluid interface (Finite Volume based methods)

- We need to follow the **evolution of the interface** $\Sigma(t)$ and the bulk phases:

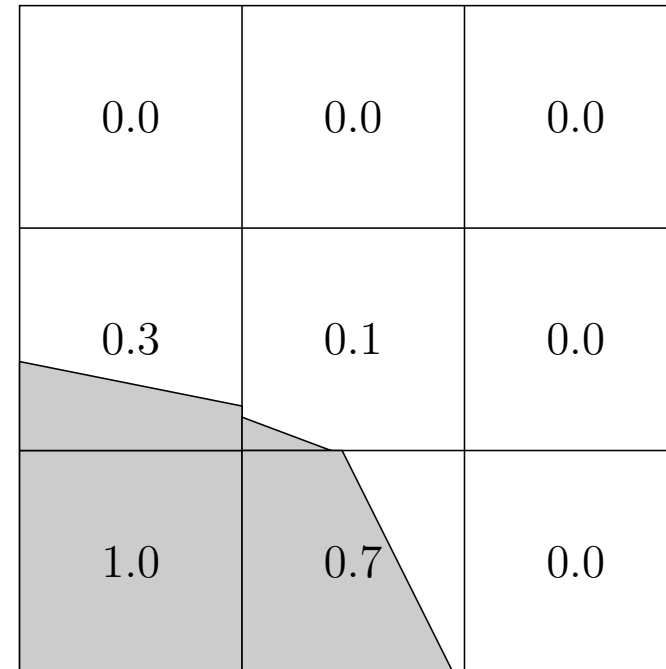
$$\Omega = \Omega_l(t) \cup \Omega_g(t) \cup \Sigma(t)$$

Arbitrary Lagrangian Eulerian (ALE) Method in OpenFoam



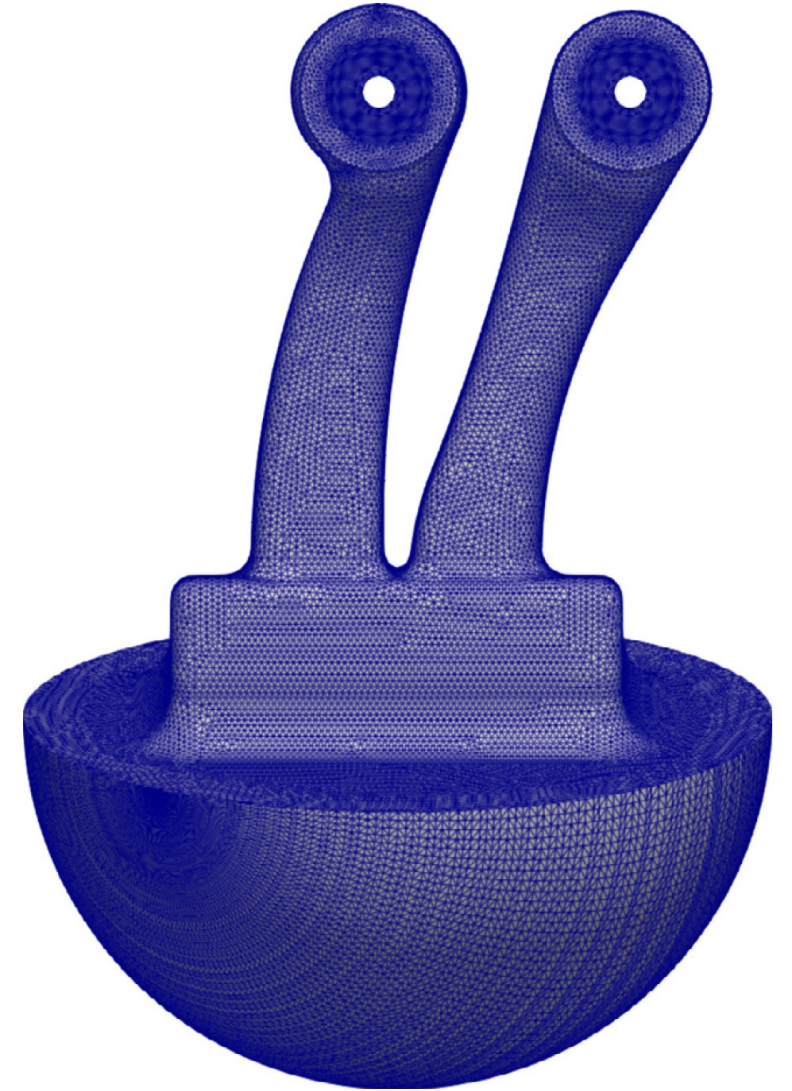
Dissertation
D. Gründing

Volume-of-Fluid Method (Eulerian)



Dissertation
M. Fricke

How to simulate two-phase flows in complex geometries?



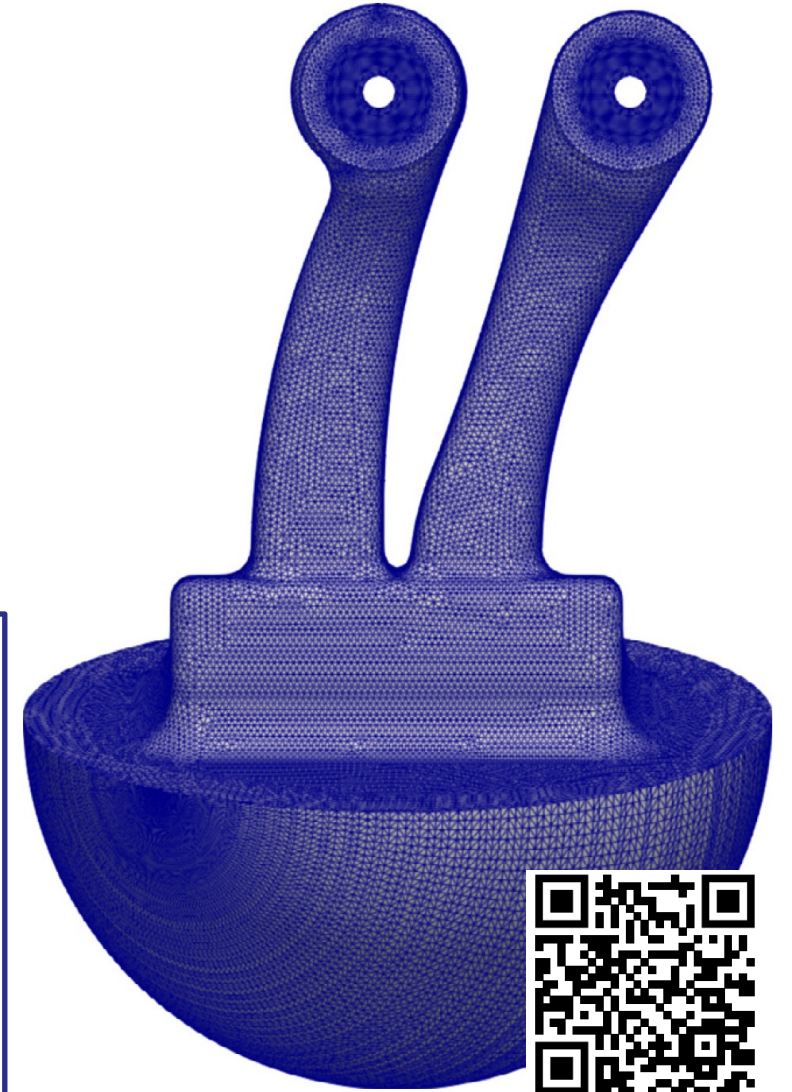
A GeoVOF method for wetting in complex geometries

The mesh matters!

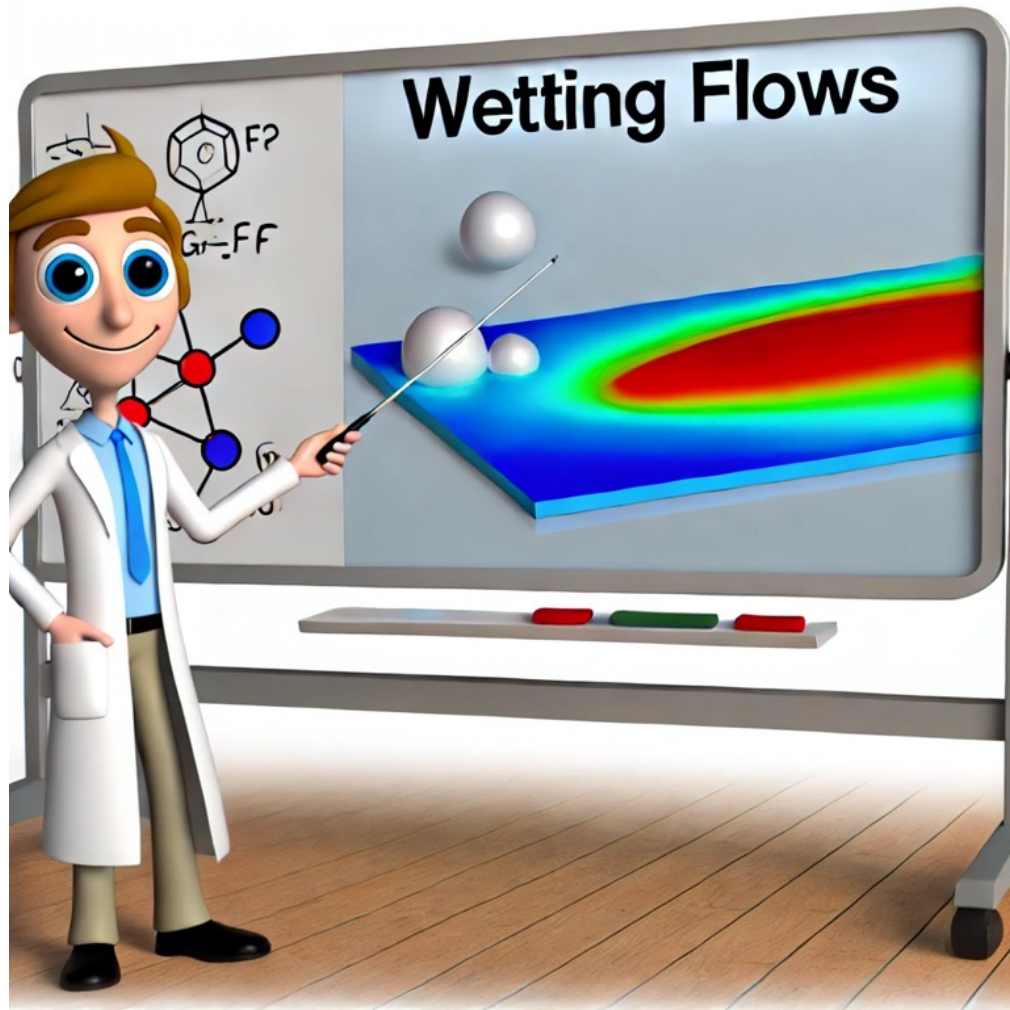


T. Maric (TU Darmstadt)

We are developing a Geometric Volume-of-Fluid on **unstructured meshes** based on the **IsoAdvect** method by J. Roenby.

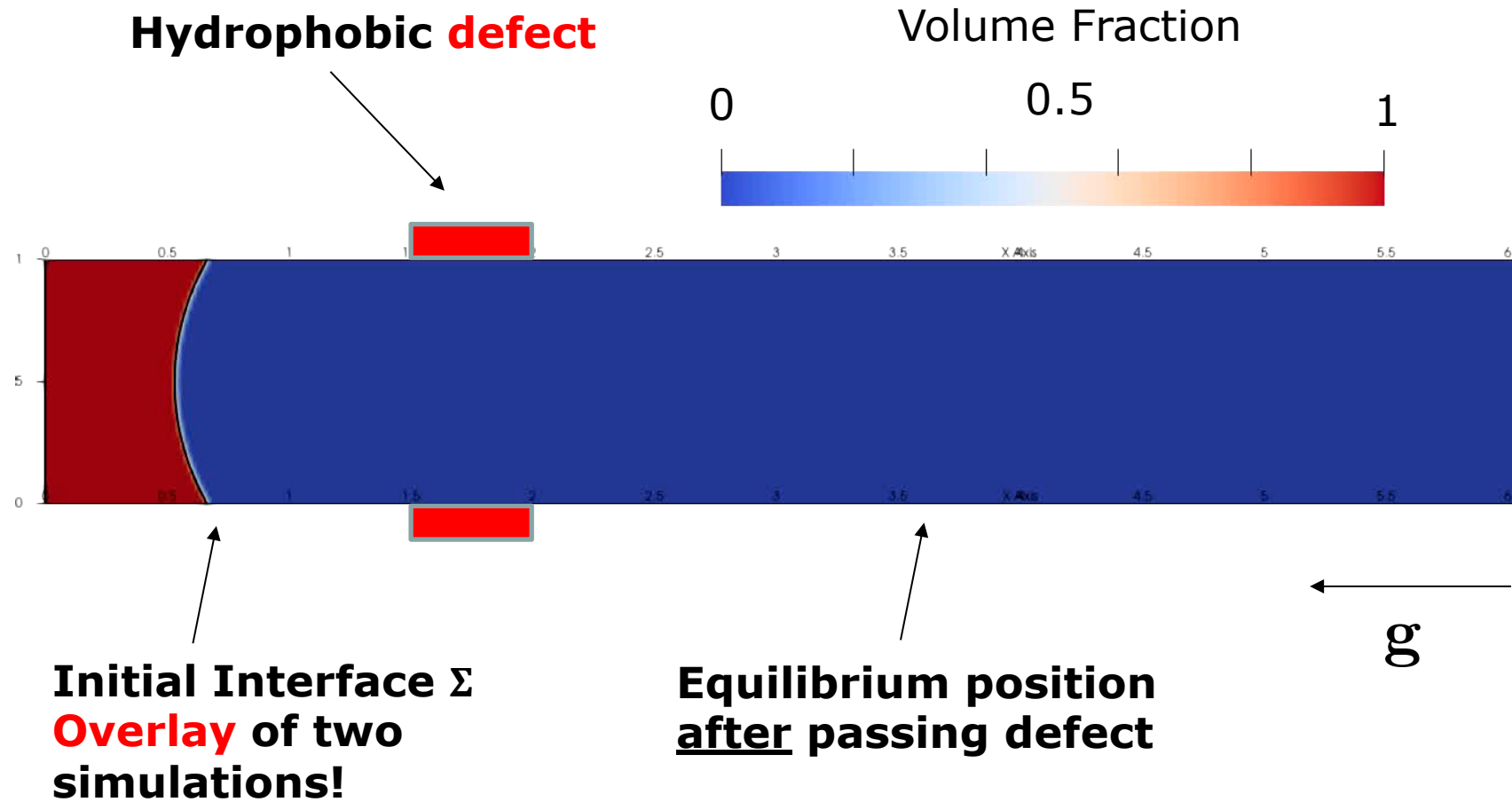


arXiv:2302.02629

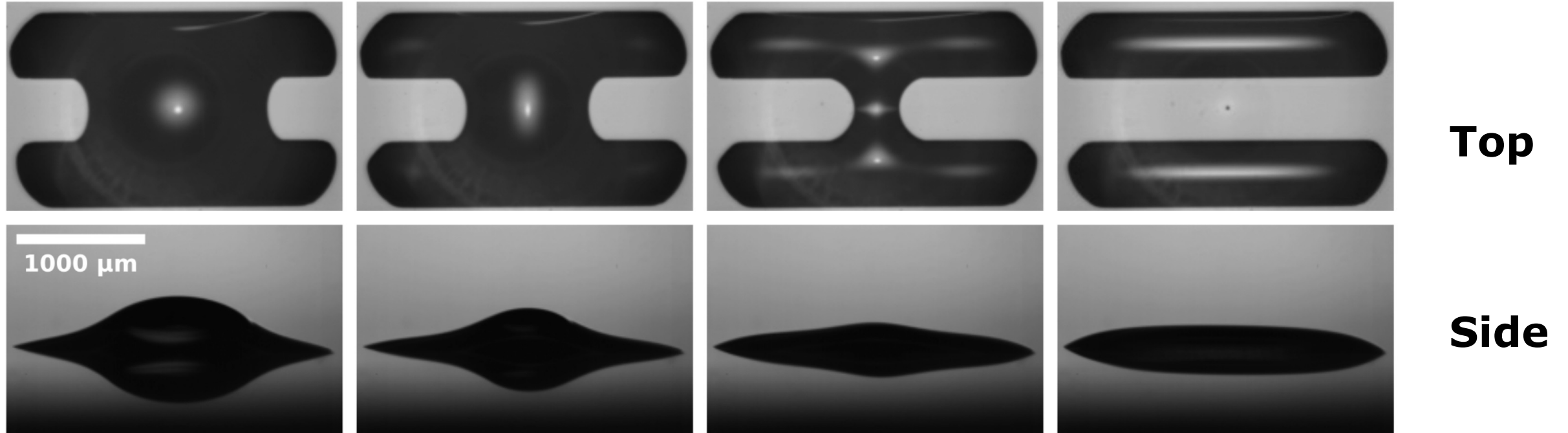


Examples: Wetting of Structured Surfaces

Example 1: Trapping of a meniscus - VOF simulations

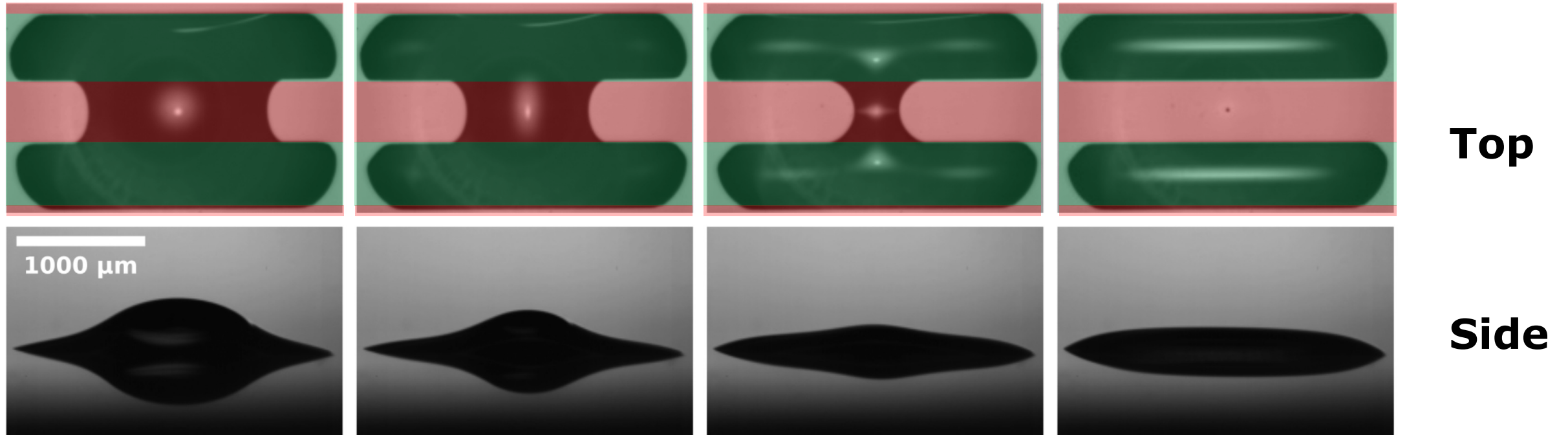


Example 2: Evaporation & Breakup on a Structured Surface



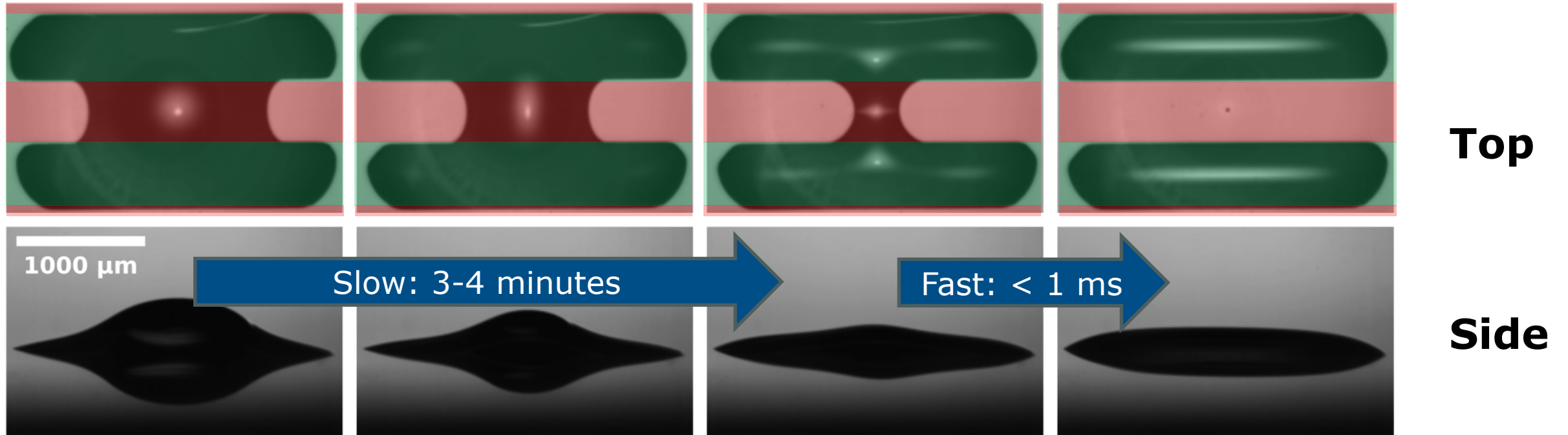
Hartmann, Fricke et al.: *Breakup dynamics of capillary bridges on hydrophobic stripes*,
International Journal of Multiphase Flow 140,
DOI: [10.1016/j.ijmultiphaseflow.2021.103582](https://doi.org/10.1016/j.ijmultiphaseflow.2021.103582), 2021

Example 2: **Hydrophobic** & **Hydrophilic** Stripes



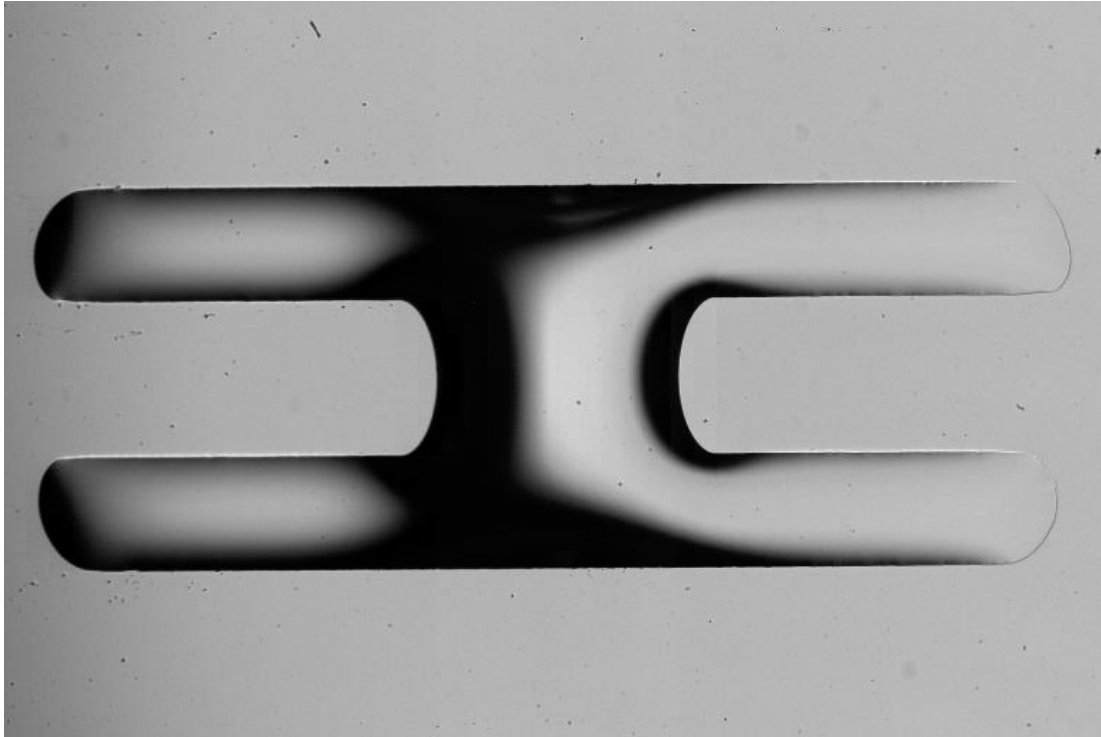
Hartmann, Fricke et al.: *Breakup dynamics of capillary bridges on hydrophobic stripes*,
International Journal of Multiphase Flow 140,
DOI: [10.1016/j.ijmultiphaseflow.2021.103582](https://doi.org/10.1016/j.ijmultiphaseflow.2021.103582), **2021**

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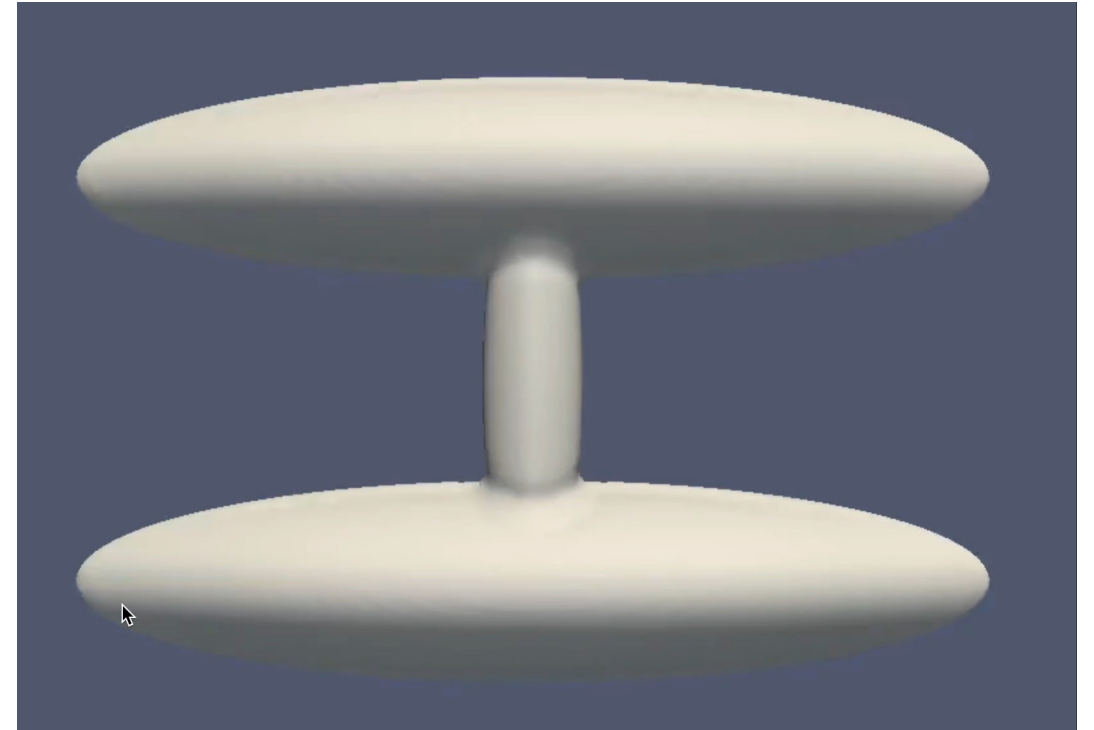


Hartmann, Fricke et al.: *Breakup dynamics of capillary bridges on hydrophobic stripes*,
International Journal of Multiphase Flow 140,
DOI: [10.1016/j.ijmultiphaseflow.2021.103582](https://doi.org/10.1016/j.ijmultiphaseflow.2021.103582), **2021**

Example 2: First naive attempts



Experiment

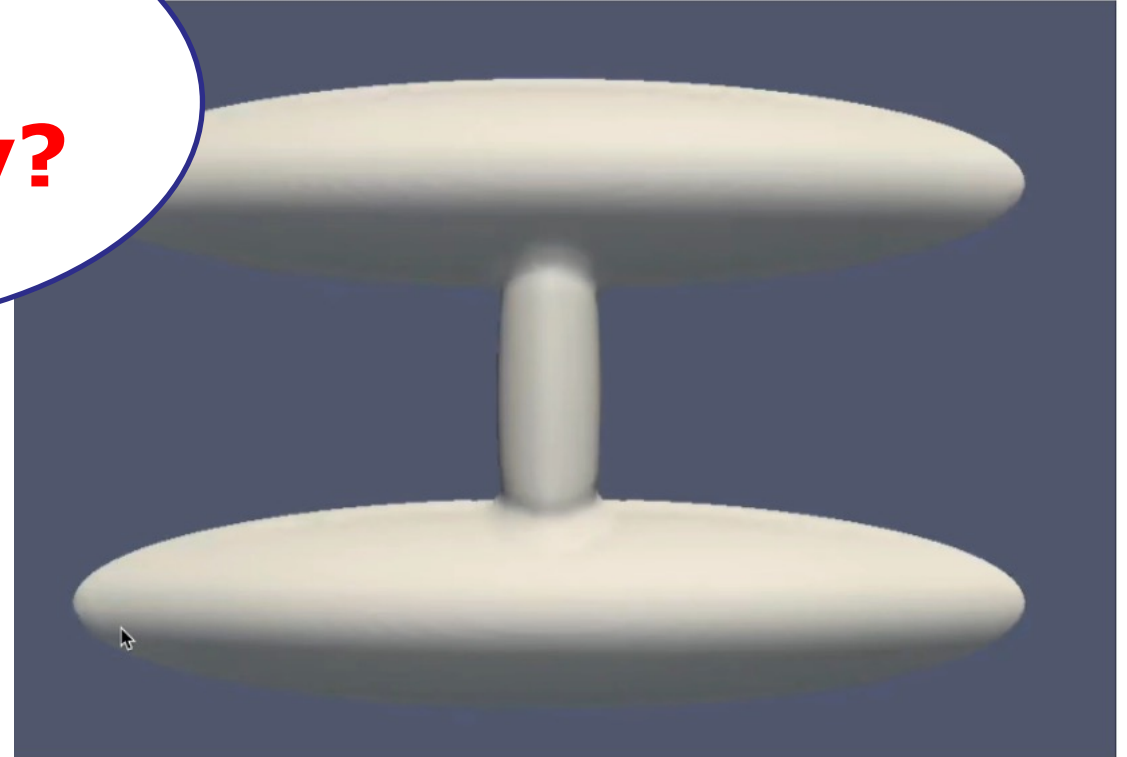


Simulation

Example 2: First naive attempts



**Initial
geometry?**

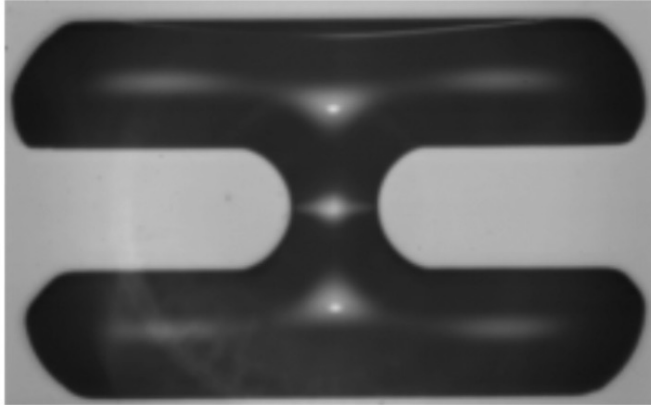


Simulation

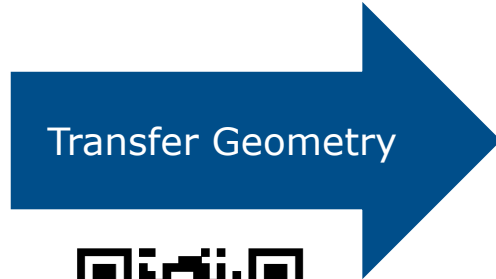
Example 2: Properly separate slow and fast



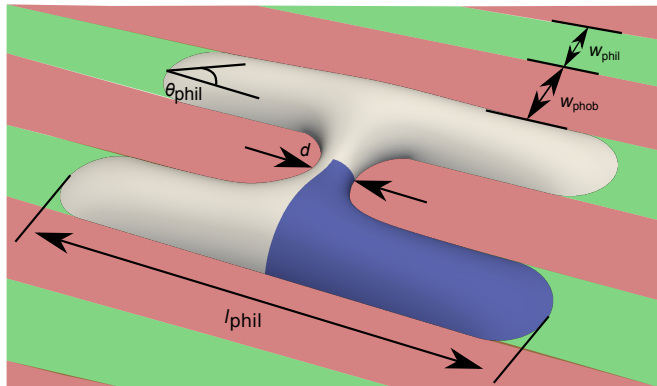
Hartmann
et. al, 2021



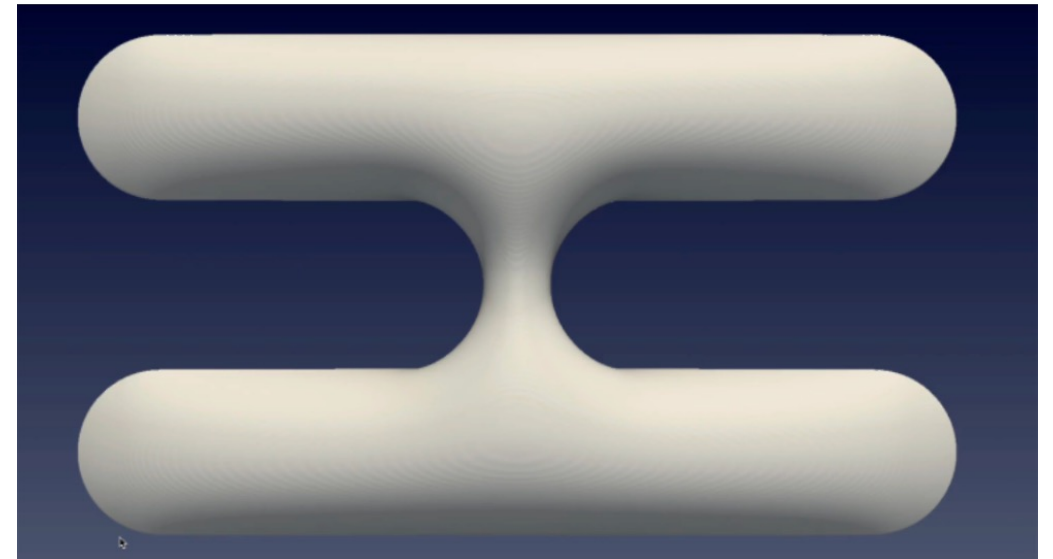
Volume-of-Fluid Two-Phase Flow



Tolle et. al,
2021



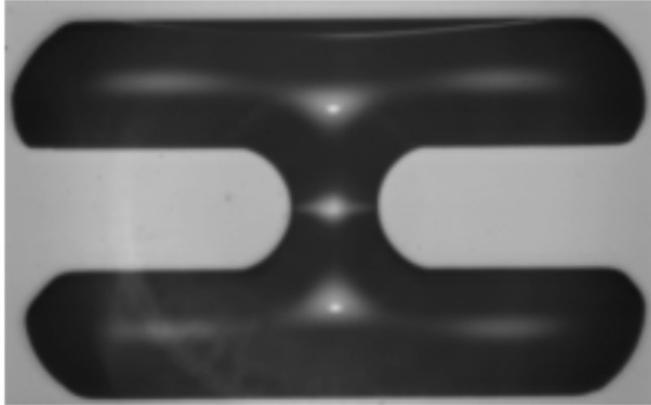
Surface Evolver until onset of breakup (find minimal surfaces)



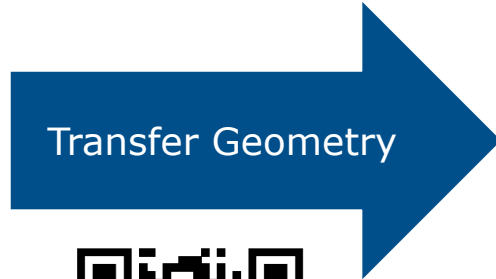
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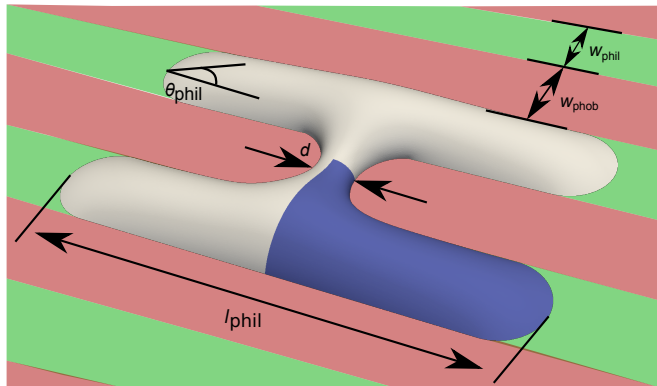
Hartmann
et. al, 2021



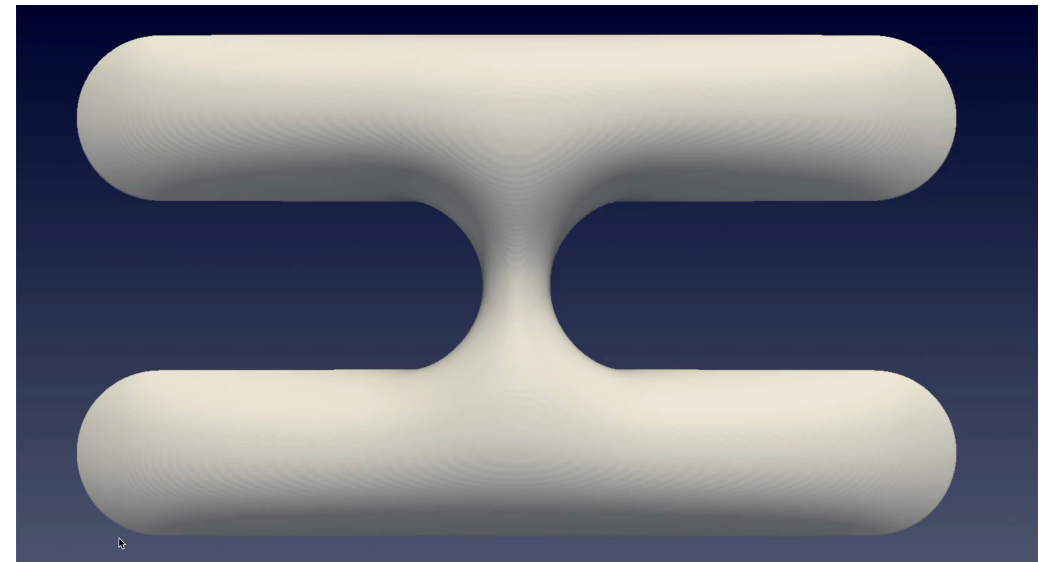
Volume-of-Fluid Two-Phase Flow



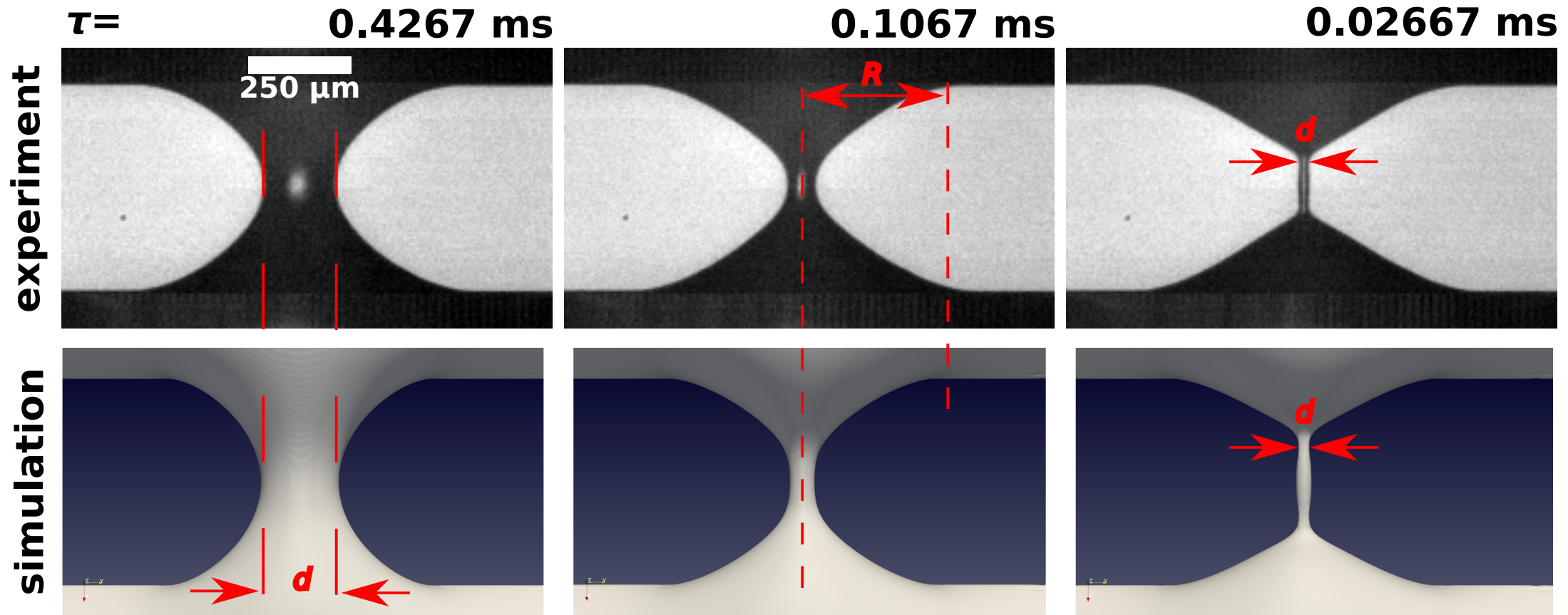
Tolle et. al,
2021



Surface Evolver until onset of breakup (find minimal surfaces)



Example 2: Problem solved!



Hartmann et. al, 2021

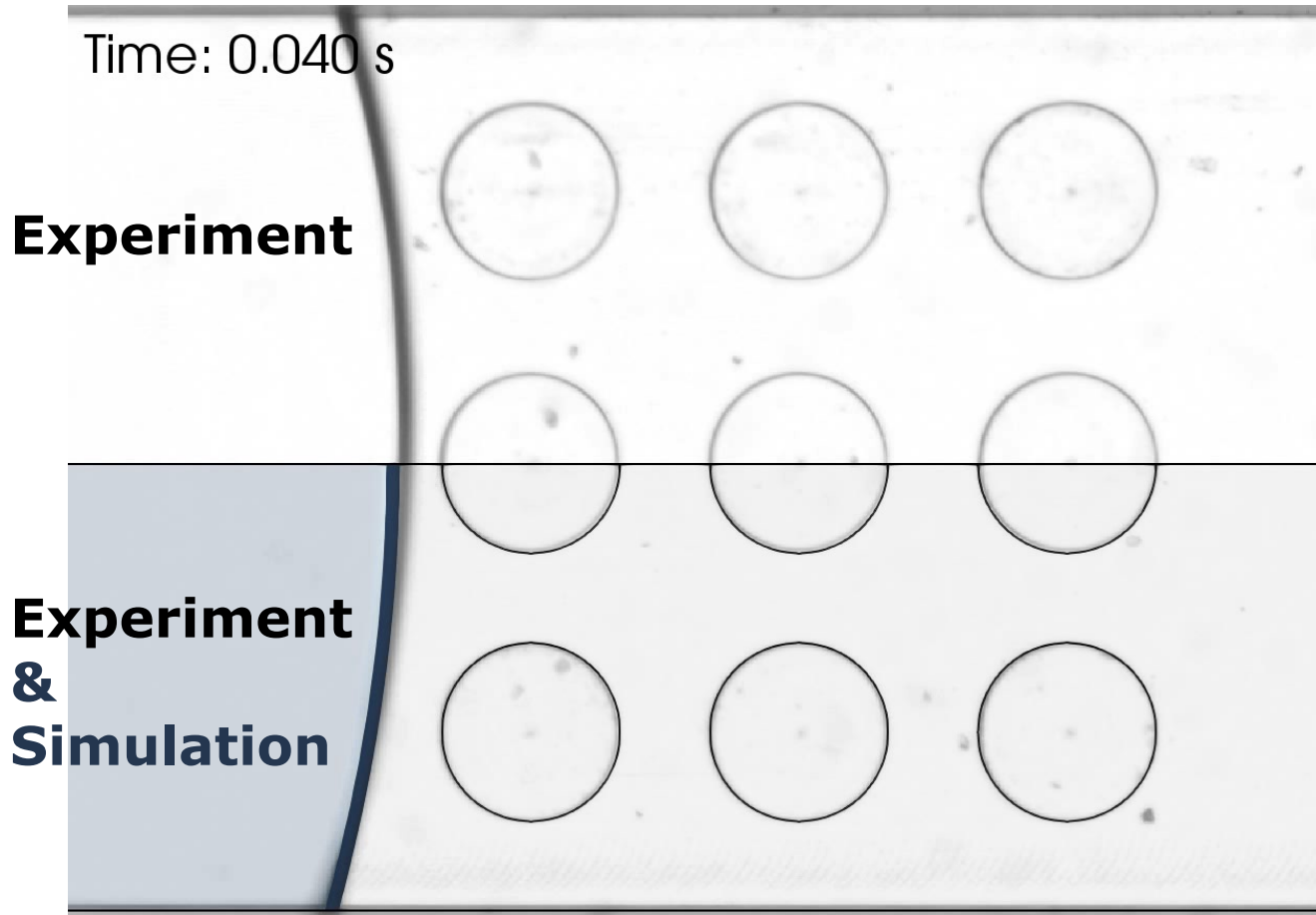


Example 3: Meniscus passing over microcavities (air-water system)

Collaboration: **Lab-on-a-chip** research @ Robert Bosch GmbH
(lead by T. Maric, TU Darmstadt)



T. Maric



- Inlet velocity 10 mm/s
- Simulations with OpenFoam using unstructured grids
- Pinning at cavity edges
- PhD work by Luise Nagel (Bosch)



arXiv:2407.18068

Thank you very much for your attention!



- Dr. Mathis Fricke
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www.sfb1194.tu-darmsatdt.de
- This work was funded by German Research
Foundation (DFG) SFB 1194 – Project ID 265191195.

List of references (I)

- M. Fricke, S. Raju, E. A. Ouro-Koura, O. Kozymka, J. De Coninck, Z. Tukovic, T. Maric, D. Bothe: *Bridging the scales in capillary rise dynamics with complexity-reduced models*, Preprint: arXiv:2311.11947 (**2023**)
- M. Fricke, C. Bernklau, E. Diehl, J. De Coninck, S. Ulbrich, D. Bothe: *On the problem of optimal fluid transport in capillaries*, Preprint: arXiv:2404.12263 (**2024**)
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